University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Spring 2013

Paper E – Contemporary Physics

Time allowed – 40 minutes each section

Instructions and Information:

- ***If you have not previously passed any of the subject areas attempt any 2 of the 6 questions***
- ***If you have previously passed one subject area attempt one of the 5 questions in the other areas.***
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Nuclear & Particle Physics

(a) Assume that the half-life of the π^0 meson is 10^{-16} s. If π^0 's are produced at a speed of 0.95c in the laboratory, how long will it take in the lab for half of the π^0 's to decay ? (20)

(b) How far will they travel in this time ? (20)

Protons are accelerated through a potential difference of $2x10^9$ Volts.

(c) What speed will they achieve ? (20)

(d) Evaluate the momentum of such a proton in GeV/c. (20)

(e) Evaluate the total energy of such a proton in GeV. (20)

Atmospheric Physics

Relative humidity is a measure of how much water vapor is present in the air (represented by the vapor pressure, e) relative to how much water vapor is required to saturate the air at a given temperature (represented by the saturation vapor pressure, e_s). This temperature, at which the moisture content in the air will saturate the air, is called the dew point.

$$RH = \frac{e}{e_s} \times 100\%$$

A large number of saturation vapor pressure equations exist to calculate the pressure of water vapor over a surface of liquid water or ice. One of the equations is called the Bolton Formula, given by:

$$e_{water} = 6.112e^{17.67\times\frac{T}{(T+243.5)}}$$

where T is temperature in ^oC, and e_{water} is in hPa.

(a) Calculate the vapor pressure of water in an air parcel at see level (pressure = 101.2 kPa) that has a temperature of 30^{o}C and a dew point of 12^{o}C. (20)

(b) Calculate the relative humidity of this air parcel. (20)

(c) Calculate the water vapor mixing ratio, r,

$$r = 0.622\frac{e_{water}}{p - e_{water}}$$

(20)

(d) If the air parcel is lifted adiabatically to 50 kPa (500mb), what are its new temperature, vapor pressure, saturature vapor pressure, and relative humidity? (hint: for adiabatic processes, potential temperature is conserved). (20)

$$\Theta = T\left(\frac{p_0}{p}\right)^{\kappa}$$

with $\kappa = 0.286$

(e) From (d), you should conclude that the parcel is supersaturated (e.g. RH > 100%). If we assure that the actual vapor pressure of the parcel (from d) is its saturation value, what then is the dewpoint temperature? (20)

Optics

A photographic camera has a lens with a 50-mm focal length. This camera is used to photograph a person of 175 cm in height, such that the image is 30 mm high in the film plane.

(a) How far from the camera should the person stand ? (20)

(b) If the lens aperture is 1 cm in diameter and using light of 500 nm, calculate the best resolution that can be obtained on the film (from an aberration-free lens). (50)

(c) Under the conditions above, estimate the smallest features that can be resolved in the person being photographed. (30)

Atomic & Molecular Physics

Rubidium has two natural isotopes: ^{85}Rb has nuclear spin 5/2 and ^{87}Rb has nuclear spin 3/2. Its ground state electronic structure is a single valence 5s electron outside a closed "shell" similar in structure to Kr. The lowest excited electronic states are 5p.

(a) For ^{87}Rb, what are the quantum numbers that best describe the 5s and 5p states ? Specifically, why are there two 5p $^2P_{1/2}$ states and four 5p $^2P_{3/2}$ states ? (25)

(b) Transitions from the ground 5s states to the 5p states are centered at two wavelengths, 780 nm and 795 nm. Which of these corresponds to the 5s $^2S_{1/2}$ $\rightarrow$ 5p $^2P_{1/2}$ transition and why ? (25)

Suppose that an atom is moving in a beam of laser light tuned to a wavelength slightly longer (e.g lower energy) than the transition from the lowest ground hyperfine state if the atom were at rest.

(c) In what direction must the atom be moving in order to absorb the laser light? In what direction does the atom recoil, and why? When the excited atom re-emits a photon, how has the atom's kinetic energy been changed, on average ? (25)

(d) For a Rb atom emitting a single photon in a transition from the 5p to the 5s state, how much kinetic energy does it have as a consequence of the recoil ? If this is equated to kT, what temperature would it correspond to ? (25)

Astrophysics

Kepler-36 is a star about which two planets are known to orbit. They were detected by the reduction in light we receive from the star when the planets transit across its disk as seen from Earth.
The mass of the "parent" star has been determined from a technique called asteroseismology, a measurement of the frequencies of the star's oscillations, to be 1.07 times the mass of the Sun, with a radius of 1.6 times the Sun's.

(a) Planet "b" orbits with a period of 13.8 days. What is the radius of its orbit compared to the Earth's orbit about the Sun ? (25)

(b) Planet "c" orbits in nearly the same plane as "b" with a period of 16.2 days. How far apart are "c" and "b" at closest approach ? (25)

(c) The two planets have radii that are "b" 0.009 and "c" 0.02 times the radius of the star. For planet "c", by what factor does the light we receive from the star decrease during a transit, and how long does a transit last ? (25)

(d) Since the two planets interact by their mutual gravitation in a way that affects the timing of the transits, their masses have also been measured. Planet "b" has a mass of 4.3 times Earth, and planet "c" has mass of 7.7 times Earth. Which of these planets has a density less than 1 gm/cm^3, and which one has a density greater than 6 gm/cm^3 ? (25)

Condensed Matter Physics

Consider vibrations in a one dimensional monatomic linear chain.

(a) Consider a one dimensional linear chain comprising N atoms with equal mass M. If we suppose the motion of the atoms is allowed only along the x-direction, how many degrees of freedom are available for the motion of the center of mass and how many for oscillations ? (10)

(b) As shown in the figure below, the position of atom n is $x_n = na + u_n(t)$, where a is the lattice constant, and $u_n(t)$ is the displacement of the atom n at the time t. The displacement of atom n at $t = 0$ is given by $u_n(t=0) = u_n^0 = e^{iqna}$, which represents the longitudinal wave along the x-direction with wave-vector q. Use periodic boundary conditions (i.e., $u_n^0 = u_{n+N}^0$) to determine the allowed values for q. (15)

(c) If the deflection $u_n(t)$ is not too large, Hooke's law is valid. In this case the force acting on the atom n due to the atom n+1 is given by $F_n = k(u_{n+1}(t) - u_n(t))$, where k is the force constant. For simplicity, considering only the interaction between nearest neighbors, the total force acting on the atom n is given by $F_n^{total} = k(u_{n+1}(t) + u_{n-1}(t) - 2u_n(t))$. If we take the Ansatz for the time dependence of the displacements, $u_n(t) = u_n^0 e^{-i\omega t}$, solve the equation of motion (i.e., Newton's second law) for $u_n(t)$ and determine the dispersion relation $\omega(q)$ for the monatomic linear chain. (20)

(d) Sketch the dispersion relation $\omega(q)$ as a function of q. Evaluate the maximum value of the frequency $\omega_{\max}$ and the corresponding minimum wavelength $\lambda_{\min}$, and prove that the dispersion relation in the long wavelength limit (i.e. $\lambda = \frac{2\pi}{q} \gg a$) is $\omega \approx \sqrt{\frac{k}{M}} qa \propto q$. (20)

(e) Evaluate an expression for the group velocity ($v_g = \frac{d\omega}{dq}$) of a wave packet for the one dimensional linear monatomic chain. Determine the maximum value of the group velocity and plot it as a function of q. (15)

(f) Find the phonon density of states ($g(\omega) = \frac{1}{\pi} \frac{1}{\left|\frac{d\omega}{dq}\right|}$) for the one dimensional monatomic linear chain, and show that the total number of modes ($L \int_0^{\omega_{\max}} g(\omega) d\omega$, where L is the length of the chain) is equal to the total number of atoms N. (20)