University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Spring 2015

Paper E – Contemporary Physics

Time allowed – 40 minutes each section

Instructions and Information:

- ***Attempt any 2 of the 6 questions***
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Nuclear & Particle Physics

(a) On a graph of atomic number (Z) versus the number of neutrons (N) indicate where you would expect stable nuclei to be found. (15)

(b) On your graph indicate (and label) the direction of a line from a parent to daughter nuclei for α decay. (10)

(c) On your graph indicate (and label) the direction of a line from parent to daughter nuclei for β^+ decay. (10)

(d) Explain why a "neutron excess" is observed for heavy nuclei. (30)

A neutron star may be thought of as a **VERY** large nucleus.

(e) Estimate the mass, in kilograms, of a fragment of neutron star material of diameter 0.5 cm. (25)

(f) Evaluate the density of this material in kg/m^3. (10)

Atmospheric Physics

In the homosphere (<50 km), the dry atmosphere is assumed to have roughly constant composition due to turbulent mixing. The composition is reported in the U.S. Standard Atmosphere as fractional composition by volume. The three largest contributors (neglecting water vapor) are:

Constituent	Molar Mass (g/mol)	Fraction by Volume
N_2	28.02	78.08 %
O_2	16	20.95 %
Ar	39.95	0.93 %

(a) Assuming an ideal gas, show that fraction by volume is equivalent to molar fraction. (10)

(b) Calculate the mean molar mass of dry air,

$$M_d = \sum_i n_i M_i / \sum_i n_i$$

where n_i is the number of moles of each constituent and M_i is the molar mass of each constituent. (15)

(c) Derive the intensive form of the Ideal Gas Law for dry air (i.e. a function of pressure (P), total volume per unit mass (α), and temperature (T)) and calculate the dry air gas constant (R_d) given a Universal Gas Constant of 8.314 J/K·mol. (15)

The troposphere extends from the surface up to a height of approximately 11 km. The U.S. Standard Atmosphere models the troposphere as dry air with a surface temperature of 288.15 K, a surface pressure of 101.3 kPa, and a constant vertical temperature gradient of −6.5 K/km.

(d) Beginning with hydrostatic equilibrium

$$\frac{\partial P}{\partial z} = -g/\alpha$$

where g is constant, find the pressure as a function of height within the troposphere. (20)

(e) Calculate the mass per unit area in a hydrostatic column extending from the surface to the top of the troposphere. (20)

(f) Calculate the mass per unit surface area of a hydrostatic column extending from the surface to the top of the atmosphere. What fraction of the total mass of the atmosphere is contained in the troposphere? (20)

Optics

One common method to analyze an optical system is the Jones' matrix method. In this approach, a ray is represented by a 2-element column $\begin{pmatrix} \rho \\ \theta \end{pmatrix}$, with ρ the ray height above and θ the ray angle to the optical axis. Optical elements along the axis are represented by 2×2 matrices, multiplied in succession in which the light passes the elements. For instance, a thin lens of focal distance f is represented by $\begin{pmatrix} 0 & 1 \\ -\frac{1}{f} & 1 \end{pmatrix}$ and an empty segment of length d along the optical axis by $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$.

(a) Using this method, find the focal length of a compound lens, made of two lenses in contact with focal lengths f_1 and f_2. (20)

(b) Using this method, find the effective focal length of a compound lens, made of two lenses with focal lengths f_1 and f_2, separated by a distance d . (30)

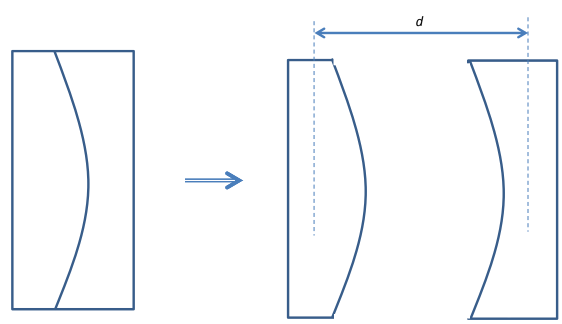
Hint: Start with a ray parallel to the axis at a distance ρ and propagate it through an appropriate sequence of empty spaces and two lenses. The angle of the exit ray is ρ divided by the effective focal length.

(c) In an example of the solution you obtained, consider a block of glass cut as in the figure below. When $d \to 0$ the combination is still a plate with an effective focal length $f = 0$ and a parallel beam incident from the left will remain parallel. The two parts are now separated by a variable distance d. For what distances d will we have convergent and divergent exit beams after the 2nd lens? Assume that the two blocks may be approximated by thin lenses. (20)

Hint: the lens maker's equation is $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where $R_{1,2}$ are the radii of curvature for the two lens surfaces.

(d) Sketch the ray diagram for a parallel incident beam and $d = f_1/2$, where f_1 is the focal length of the first lens. (15)

(e) Sketch the ray diagram for a parallel incident beam and $d = 2f_1$. (15)



Atomic & Molecular Physics

Frequency Comb

A laser frequency comb is a unique source of coherent light consisting of evenly spaced frequencies such that $f_n = f_0 + n\, f_r$ where f_r is the frequency spacing and f_0 is an offset.

(a) Suppose that two beams of light of different frequencies f_a and f_b combine in a medium that absorbs some of the light and produces a signal of some sort which can be detected and measured in the laboratory. If the material responds linearly to the incident light, what frequencies will we observe in the signal? (20)

(b) What frequencies will we observe if the material responds non-linearly, and particularly if the two frequencies differ by only a relatively small amount? (20)

(c) The time and frequency domains of laser light are closely connected. Suppose that a laser is a pulse of red light (wavelength ~600 nm) lasting for 10 nanoseconds (10^{-8} seconds) with an exponential decay of amplitude. What would the spectrum (in frequency and wavelength) of this light be? What happens to the spectrum if instead of 10 nanoseconds the pulse is only 10 femtoseconds (10^{-14} seconds)? (20)

(d) A "mode-locked" laser is one where the gain only applies for pulses of light, and femtosecond duration pulses may be obtained this way. In a "mode-locked" cavity, the pulses have a consistent phase to one another, and thus may interfere with one another when propagating through a non-linear medium. Assume that the pulses repeat with a frequency f_r. Show that a train of thousands of such pulses would produce a comb of frequencies f_n. (20)

(e) The second, our standard unit of time, is defined by the properties of an atom:

"*The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition of two hyperfine levels of the ground state of the cesium 133 atom*."

That is, this oscillator is about 9.2 GHz and can be measured with modern electronics. By contrast, the frequency of visible light is too high to measure electronically. To connect these two regimes, a laser frequency comb spanning many octaves (factors of 2) would be needed. Consider just one of these factors of 2.
Without an external reference (i.e. the light is "self-referencing"), how could we find f_0 and tell that the comb's f_r is such that on of its frequencies falls exactly on twice the reference frequency? (Hint: Consider your answer to part b.) This is the basis for an optical atomic clock. (20)

Astrophysics

Solar Sailing

Consider a spacecraft with an extended surface that we can think of as a "sail" exposed to light from the Sun. The solar constant, the irradiance of the Earth with light from the Sun, has been measured by Earth satellites to be about 1.36 kilowatts per square meter above the atmosphere at Earth's distance from the Sun (1 astronomical unit or 1.5×10^8 km). Sunlight exerts a pressure on the sail due to the momentum carried by photons, and the pressure results in a force and acceleration. The force due to the wind of particles from the Sun is several orders of magnitude smaller and should be neglected in this problem.

Speed of light c = 299,792,458 m/s

(a) If the "sail" were perfectly absorbing, what is the magnitude and direction of the force exerted on the spacecraft by the pressure of sunlight on its sail, assuming a perfectly absorbing black area of 100x100 meters extended perpendicular to a line to the Sun. (25)

(b) What would this be if the surface were perfectly reflecting? (25)

(c) Since the spacecraft would be in orbit about the Sun, consider only the effect of light pressure on the radial motion assuming that the sail will turn as needed to always face the Sun. How would the radial acceleration depend on R, the distance of the spacecraft from the Sun? Compare the force of gravity on the spacecraft pulling toward the Sun to the force of light pressure pushing it away for a perfectly reflecting sail 100x100 meters and mass 100 kg. (25)

(d) If the spacecraft with mass 100 kg started out at 0.1 astronomical unit in a circular orbit, the radial pressure would change the radial velocity v_r and the spacecraft would accelerate away from the Sun. Considering only its radial motion, what would its speed be when it reached into the Oort Cloud, say 10,000 astronomical units from the Sun? Will the spacecraft escape the solar system? (25)

Condensed Matter Physics

This problem concerns the tight-binding energies of a crystal with the hexagonal Bravais lattice symmetry. The α-orbital tight-binding energy $E_\alpha\left(\vec{k}\right)$ is given by $E_\alpha(\vec{k}) = \varepsilon_\alpha - J_\alpha^0 - \sum_n J_\alpha^1(\vec{R}_n)e^{-i\vec{k}\cdot\vec{R}_n}$, where α represents the atomic orbital, ε_α is the atomic orbital energy (e.g., ε_s , the s-orbital energy, ε_p , the p-orbital energy, etc.), J_α^0, the on-site potential energy, $J_\alpha^1(\vec{R}_s)$, the off-site potential energy, $\vec{k}$, the reciprocal vector, and $\vec{R}_n$, the nearest neighbor lattice vectors. For a given orbital α, ε_α and J_α^0 are constants, and the tight-binding energies $E_\alpha\left(\vec{k}\right)$ will depend on the symmetry of the crystal via the third term in the equation.

(*To answer this problem it is not critical that you understand exactly what is meant by the tight-binding energy of a crystal. What you need to do in this problem is to know how to use it to study the crystal with the hexagonal lattice symmetry. The basic knowledge that you need is the hexagonal symmetry, the primitive lattice (or basis) vectors, the nearest neighbor lattice vectors, and the reciprocal lattice vectors.)*

(a) Fig. 1 shows the simple cubic (sc) Bravais lattice. The solid circles represent points (or atoms) forming the lattice. Write down their primitive lattice vectors ($\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$) (i.e., the bold arrows shown in Fig. 1) in terms of the lattice constant a and the unit vectors ($\hat{i}$, $\hat{j}$, and $\hat{k}$) in Cartesian coordinates. Namely, you need to find the x-, y-, and z-components for each vector, and express it as $\vec{a}_i = a_{ix}\hat{i} + a_{iy}\hat{j} + a_{iz}\hat{k})$, i=1, 2, 3. (20)

(b) Find the coordinates of the 6 nearest neighbor lattice vectors $\vec{R}_n = (R_{nx}, R_{ny}, R_{nz})$, n = 1, 2,…,6, for the sc lattice with respect to the origin. Namely, you need to find the components of the vectors which start from the origin and end on the nearest neighbor points in the sc lattice. Keep in mind that some of nearest neighbor lattice vectors might end on the points in the nearest neighbor cubes of the single cube which are not shown in Fig.1. (20)

(c) Use the formula given at the first paragraph of the problem and the results from (b) to express the s-orbital tight binding energy $E_\alpha\left(\vec{k}\right)$ in the sc as a function of $\vec{k}$, ε_α and J_α^0

(Hint: $J_s^1(\vec{R}_n) = J_s^1$ is independent of $\vec{R}_n$ when the orbital has a spherical symmetry, like s-orbital, and ε_α and J_α^0 are constants) (20)

(d) Using the results from (c) find the s-orbital tight binding energy $E_\alpha\left(\vec{k}\right)$ for the sc in terms of ε_α, J_α^0 , and J_s^1 at Γ point (i.e., $\vec{k} = (0,0,0)$) and R point (i.e., $\vec{k} = \frac{\pi}{a}(\frac{1}{2},\frac{1}{2},\frac{1}{2})$, respectively. (20)

(e) Calculate the effective mass with respect to $E_s\left(\vec{k}\right)$ for the fcc which is defined as $m_x^* = \dfrac{\hbar^2}{\dfrac{\partial^2 E}{\partial k_x^2}}$; $m_y^* = \dfrac{\hbar^2}{\dfrac{\partial^2 E}{\partial k_y^2}}$; $m_z^* = \dfrac{\hbar^2}{\dfrac{\partial^2 E}{\partial k_z^2}}$. What is the effective mass at Γ point (i.e.,) $\vec{k} = (0,0,0)$? (20)

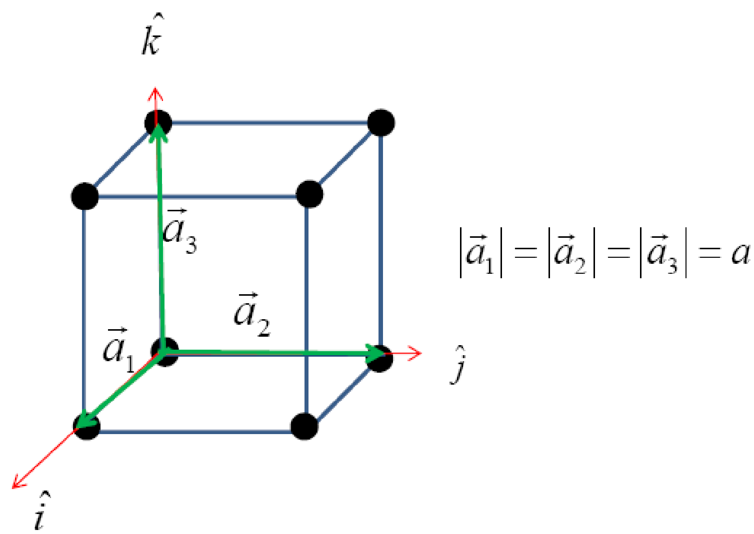


Fig. 1 The simple cubic (sc) Bravais lattice. The balls represent the points (or atoms) forming the lattice. The bold arrows are the primitive lattice vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$. The thin black arrows denote the unit vectors of the Cartesian coordinates. The origin is located at the corner of the single cube.