University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Spring 2015

Paper D – Quantum Mechanics

Time allowed – 90 minutes

Instructions and Information:

- Answer both questions
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Quantum Mechanics Basic Level

Planck's black body energy density distribution is given by $u(\nu,T)=\dfrac{8\pi\nu^2}{c^3}\dfrac{h\nu}{e^{h\nu/k_BT}-1}$:

(a) Show that it reduces to the Rayleigh-Jeans's energy density distribution $u_{RJ}(\nu,T)=\dfrac{8\pi\nu^2}{c^3}k_BT$ in the low frequency range. (17)

(b) Show that it reduces to the Wien energy density distribution $u_W(\nu,T)=A\nu^3e^{-\beta\nu/T}$ in the high frequency range. In this case, obtain expressions for the parameters ***A*** and ***β*** in terms of the physical constants, h, c and k_B (18)

Quantum Mechanics Intermediate Level

A particle in an infinite square well has as its initial wave function an even mixture of the first two stationary states: $\Psi(x,0) = A[\psi_1(x)+\psi_2(x)]$, where

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), & \text{if } 0 \le x \le a, \\ 0, & \text{otherwise} \end{cases}.$$

(a) Normalize $\Psi(x,0)$. (That is, find **A**. This is very easy, if you exploit the orthonormality of ψ_1 and ψ_2.) (10)

(b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time. To simplify the result, let $\omega \equiv \pi^2\hbar/2ma^2$. (15)

(c) Compute $\langle x \rangle$ in the state of $\Psi(x,t)$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (15)

(d) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? (10)

(e) Find the expectation value of $\hat{H}$. How does it compare with the energies $\mathbf{E_1}$ and $\mathbf{E_2}$ from part (d)? (15)

(**Hint**: $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$ and $\sin(x)\sin(y) = \frac{1}{2}(\cos(x-y)-\cos(x+y))$).