Pre-lab Quiz/PHYS 224
Thin Lens and Image Formation

Your name________________________ Lab section________________________

1. What do you investigate in this lab?

2. The focal length of a bi-convex thin lens is 10 cm. To a real image with magnification of \(-2\), what is the object distance \(p\)? What is the image distance \(q\)? Is the image upright or inverted? (Answer: \(p=15\) cm, \(q=30\) cm, inverted)

3. The focal length of a bi-convex thin lens is 10 cm. The focal length of a convex-concave thin lens is \(-20\) cm. If placing them in parallel and in close contact, what is the compound focal length? (Answer: 20 cm)

4. For a red light beam, the refractive index of a thin lens is 1.510. For a blue light beam, the refractive index is 1.525. For the red light beam, the focal length of the thin lens is 20.0 cm. For the blue light beam, what is the corresponding focal length? (Answer: 19.4 cm)
Lab Report/PHYS 224

Thin Lens and Image Formation

Name_________________________ Lab section_____________________

Objective
In this lab, you will measure the focal length of thin lens, use thin lens to form image and study the thin-lens equation, and observe the chromatic aberration of thin lens.

Background
Thin-lens equation Optics employs a variety of lenses for image formation and spectroscopic measurement. In this lab, you will study how images are formed through diffraction by thin lenses. You will use two different kinds of thin lenses (Figure 1): two bi-convex lenses (converging) and a convex-concave lens (diverging).

A lens is characterized by its focal points (F, F’) and focal length (f). For a converging lens, if the incident rays are parallel with its principle axis, the refracted rays by the lens converges at the focal point on the same side of the lens as the refracted rays. For a diverging lens, if the incident rays are parallel with its principle axis, the refracted rays, when traced backward, converge at the focal point on the same side of the lens as the incident rays. The focal length, f, is thus positive for a converging lens and is negative for a diverging lens; the magnitude of f is the distance between the focal point and the center of the lens.

As shown in Figure 2, if an object is placed near the principle axis of a thin lens, the formed image through diffraction by the thin lens is described by the thin-lens equation:

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]  

(1)

The object distance, p, is the distance between the object and the center of the lens. The image distance, q, is the distance between the image and the center of the lens; q is positive for a real image and negative for a virtual image. The magnification of the image is:

\[ M = \frac{image \ height}{object \ height} = -\frac{q}{p} \]  

(2)

The image height is positive for an upright image and negative for an inverted image.

Place two thin lenses (respectively with the focal length of \( f_1 \) and \( f_2 \)) together such that their principle axes coincide (so they are parallel) and they are in contact. This combination of thin lenses behaves also like a thin lens. Using Equation (1), it is straightforward to derive the compound focal length as
Converging lens Because a converging lens has positive focal length, following Equation (1) a converging lens can form both real and virtual images. If the object distance \(p<f\), Equation (1) leads to a negative \(q\) inferring a virtual image. If the object distance \(p>f\), Equation (1) leads to a positive \(q\) inferring a real image. Real images can directly project on a screen, enabling direct measurement of \(q\) and \(M\). Thus, selecting an appropriate \(p\), one can form a real image, measure \(q\), and use Equation (1) to calculate \(f\).

Method 1 Placing an object far away from the lens, \(p\) is much longer than \(f\). Equation (1) leads to \(q\approx f\). Thus, a real image forms approximately at the focal point \(F'\). Placing an object near \(F\) with \(p>f\) but \(p=f\), Equation (1) leads to a very large \(q\). Thus, a very large real image forms on a screen placed far away. These are simple methods to approximately measure the focal length.

Method 2 Placing the object at \(p=3f\), Equation (1) leads to that \(q=1.5f\).

Method 3 Placing the object at \(p=2f\), Equation (1) leads to that \(q=2f\).

Diverging lens Because the focal length of a diverging lens is negative, following Equation (1), it can form only virtual images which cannot directly project on screen. According to Equation (3), if combing a diverging lens with a converging lens with a shorter focal length, the compound focal length is however positive and can thus be measured directly. The focal length of the diverging lens can then be extracted.

Chromatic aberration The focal length of a bi-convex thin lens placed in air follows the lens-maker’s equation

\[
\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).
\]

\(n\) is the refractive index of the lens. \(R_1\) is the radius of the curvature of its front surface and \(R_2\) is the radius of the curvature of its back surface. Because the refractive index is different at different wavelengths, the focal length varies with the wavelength. For example, \(n\) is smaller for red light than for blue light. Thus, the focal length is longer for red light than for blue light, leading to the chromatic aberration for the lens.
EXPERIMENT

In this lab, the object, lens holder, and screen are placed on track and can move along the track. \( p \) and \( q \) are determined by reading their positions from the attached rule on the track.

Procedures

1. Measure the object size
   Use the calibrator to measure the object height. Recommend to measure the height of the arrow, \( h \). Record it in Table 1.

2. Measure the focal length of the first bi-convex lens
   (Method 1) Place the lens on the lens holder. Move the screen to the opposite end of the track. Move the lens holder until you can see the sharpest image on the screen. Using the rule, determine the image distance between the lens holder and the screen which is approximately the focal length, \( f_1 \). Record it in Table 1. Move the lens holder towards the object; when their distance is about \( f_1 \), you should see a very large and sharp image on the screen.

   (Method 2) Move the lens holder away from the object to make \( p \approx 3f_1 \). Then, move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen. Record it in Table 1. Use the calibrator to measure the height of the arrow on the screen, \( h' \). Be careful with the sign of \( h' \).

   (Method 3) Move the lens holder towards the object to make \( p \approx 2f_1 \). Then, move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen. Record it in Table 1. Use the calibrator to measure the height of the arrow on the screen.

   (Method 4) Move the lens holder towards the object to make \( p \approx 1.5f_1 \). Then, move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen. Record it in Table 1. Use the calibrator to measure the height of the arrow on the screen.

Using Equation (1) and the \( p \) and \( q \) data from Methods 2-4, calculate

<table>
<thead>
<tr>
<th>( p ) (cm)</th>
<th>( q ) (cm)</th>
<th>( f_1 ) (cm)</th>
<th>( M = -q/p )</th>
<th>object height</th>
<th>image height</th>
<th>( M = h'/h )</th>
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<td>(( \sim 1.5f_1 ))</td>
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</table>

Using Equation (1) and the \( p \) and \( q \) data from Methods 2-4, calculate the corresponding focal length and record in Table 1. Calculate their mean value and record it here:

\[
\bar{f}_1 =
\]

Using the \( p \) and \( q \) data from Methods 2-4, calculate \( M = -q/p \) and record in Table 1.
Using the \( h \) and \( h' \) data from Methods 2-4, calculate \( M = h'/h \) and record in Table 1.
3. Measure the chromatic aberration of the first bi-convex lens
Move the lens holder towards the object to make \( p \approx 1.5f_1 \). Now, place the red filter on the object, and move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen. Record it in Table 2.

Now, place the blue filter on the object, and move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen. Record it in Table 2.

Using Equation (1) and the \( p \) and \( q \) data, calculate the corresponding focal length and record in Table 2.

| TABLE 2  Chromatic Aberration |
|-----------------------------|--|------------------|
| p (cm) | q (cm) | f (cm) |
| Red filter | | |
| Blue filter | | |

4. Measure the focal length of the Plano-convex lens (converging)
Place the lens on the lens holder. Move the object and the screen to the opposite ends of the rule. Move the lens holder until you can see the sharpest image on the screen. Using the rule, determine the image distance between the lens holder and the screen which is approximately the focal length, \( f_2 \). \((\text{Method 3})\) Move the lens holder towards the object to make \( p \approx 2f_2 \). Then, move the screen until you can see the sharpest image on the screen. Using the rule, determine the image distance \( q \) between the lens holder and the screen.

Record \( p = q = \)

Using Equation (1) and the \( p \) and \( q \) data calculate the focal length \( f_2 = \)

5. Measure the compound focal length for the two converging lenses
Carefully place the bi-convex lens and the plano-convex lens (in contact and in parallel) on the lens holder. Move the lens holder towards the object to make \( p \approx \frac{2f_1f_2}{f_1 + f_2} \). Then, move the screen until seeing the sharpest image. Determine the image distance \( q \) between the lens holder and the screen.

Record \( p = q = \)

Using Equation (1) and the \( p \) and \( q \) data, calculate the compound focal length \( f_{1,2} = \)

6. Measure the compound focal length of the bi-convex lens & the plano-concave (diverging) lens
Place the bi-convex lens and the plano-concave (in contact and in parallel) on the lens holder. Move the lens holder towards the object until \( p \approx 3f_1 \). Then, move the screen until seeing the sharpest image. Determine the image distance \( q \) between the lens holder and the screen.

Record \( p = q = \)

Using Equation (1), calculate the compound focal length: \( f_{1,3} = \)

Using Equation (3), \( f_1 \), and \( f_{1,3} \), calculate the focal length of the diverging lens: \( f_3 = \)
Questions

1. In procedure 2, are all the images upright or inverted? What is the direction of the horizontal arrow on the screen?

2. Using the first bi-convex lens to form a magnified real image, in what range should the object distance be?

3. Using Equation (4) and the data in Table 2, calculate the ratio of the refractive index of the lens between the red light and the blue light.

4. Calculate \( \frac{f_1 f_2}{f_1 + f_2} \) and compare it with the measured \( f_{1&2} \). Calculate their difference in percentage. Will \( f_{1&2} \) change if you switch the position of the two lenses in the combination?

5. For the combination of the diverging lens and the converging lens, what is the compound focal length? Can you use this combination to measure the focal length of the diverging lens?