1. What do you investigate in this lab?

2. For a pair of Helmholtz coils described in this manual and shown in Figure 2, \( r = 0.105 \text{ m}, N = 130, I = 0.4 \text{ A}, \) what is the magnitude of the produced magnetic field at the midway of the Helmholtz coils? (Answer: \( B_{helm} = 4.46 \times 10^{-4} \text{ tesla} = 4.46 \text{ gauss} \))
Objective

In this lab, you measure Earth’s horizontal magnetic field.

Background

(A) “Magnetic pendulum”: a bar magnet swings in magnetic field

As shown in Figure 1, when a bar magnet is placed in a magnetic field \( B \), the net magnetic force on the bar magnet is zero. However, the net torque \( \tau \) on the bar magnet is zero only when the bar magnet is aligned with \( B \) (Figure 1(b)). For quantitative description, one visualize the bar magnet as a magnetic dipole moment \( \mu \). Note that \( \mu \), \( B \), and \( \tau \) are vector quantities and thus have directions. The direction of \( \mu \) points from the south pole to the north pole. The magnitude of \( \tau \) is given by:

\[
\tau = \mu B \sin \theta ,
\]

where \( \theta \) is the angle between \( \mu \) and \( B \). When the bar magnet is not aligned with \( B \) (namely, \( \mu \) and \( B \) are not parallel with each other), \( \tau \) becomes non-zero and tends to rotate the bar magnet to be aligned with \( B \) (Figure 1(a) and Figure 1(c)). Denote \( I_{\text{inertial}} \) as the moment of inertia for the bar magnet and \( \alpha \) as the angular acceleration of the bar magnet. Then, \( I_{\text{inertial}} \alpha \) equals the torque:

\[
I_{\text{inertial}} \alpha = -\tau = -\mu B \sin \theta , \quad \text{or} \quad \alpha = -\frac{\mu B}{I_{\text{inertial}}} \sin \theta .
\]

For small angle, \( \sin \theta \approx \theta \), and Equation (2) becomes

\[
\alpha = -\frac{\mu B}{I_{\text{inertial}}} \theta .
\]

Equations (2) and (3) resemble those describing the motion of pendulum. For the pendulum, the
torque is induced by gravity, making the pendulum swing back-and-forth about the equilibrium position. At small angle, its motion is simple harmonic motion. Following the discussion of pendulum in PHYS 221, the small-angle back-and-forth swing of the bar magnet is also simple harmonic oscillation with period $T$ given by,

$$T = 2\pi \sqrt{\frac{I_{\text{inertial}}}{\mu B}}. \quad (4)$$

Rewrite Equation (4) as

$$B = \left(\frac{4\pi^2 I_{\text{inertial}}}{\mu}\right) \frac{1}{T^2}. \quad (5)$$

Using Equation (5), we can first determine the parameter $\frac{I_{\text{inertial}}}{\mu}$ of the bar magnet by measuring $1/T^2$ for the bar magnet in a known magnetic field, such as $B_{\text{helm}}$ produced by Helmholtz coils. Afterwards, we can determine the magnitude of an unknown magnetic field, such as the Earth’s magnetic field, by measuring $1/T^2$ for the bar magnet in this field.

(B) Earth’s magnetic field: Earth’s magnetic field can be viewed roughly as induced by a huge bar magnet at Earth’s center pointing along Earth’s magnetic axis which changes very slowly. At present, the geomagnetic South Pole is near the geographic North Pole and the geomagnetic North Pole is near the geographic South Pole. Like the magnetic field produced by a bar magnet, the magnetic field on the Earth surface varies from one location to another. At each location, the geomagnetic field has a horizontal component ($B_{\text{earth},h}$) tangent to the Earth surface and point to the geomagnetic north pole, and a vertical component. At a location, the magnitude of $B_{\text{earth},h}$ depends on such as its geomagnetic latitude (not identical to geographic latitude), thus varying from one location to another. At any location, the magnitude and the orientation of $B_{\text{earth},h}$ may be affected also by the magnetic materials in the surrounding area. Therefore, it is important to map out the magnitude and the orientation of $B_{\text{earth},h}$ on earth surface.

(C) The magnetic field of Helmholtz Coils: Figure 2 displays a pair of Helmholtz coils consisting of two identical circular coils placed directly facing each other, such that the axial axis, which connect their centers, is perpendicular to both coils. The two coils have the same radius $r$ and are separated by distance $r$. Each coil is wound with $N$ turns of wire. When both coils carry current in the same direction with the same magnitude $I$, very uniform magnetic field is produced between the two coils. Specifically, at the midway (Point C) between the coils, the magnetic field $B_{\text{helm}}$ is along the axial axis with the magnitude given by:

$$B_{\text{helm}} = \left(\frac{4}{5}\right)^{3/2} \left(\frac{\mu_0 NI}{r}\right), \quad \text{where } \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A}. \quad (6)$$

(D) Measure the unknown Earth’s magnetic field using a “magnetic pendulum” and the known magnetic field of Helmholtz coils: Align a pair of Helmholtz coils such that at its midway $B_{\text{helm}}$ is parallel with $B_{\text{earth},h}$, and the resultant horizontal component is
\[ B_{res} = B_{\text{earth},h} + B_{\text{helm}} = B_{\text{earth},h} + \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N I}{r}. \] (7)

A bar magnet is placed at the mid-way of the Helmholtz coils and is restricted only to horizontal rotation. If the magnet torque dominates all other torques, the bar magnet undergoes simple harmonic horizontal small-angle swing. Following Equation (5), its time period is

\[ \frac{1}{T^2} = \frac{\mu}{4\pi^2 I_{\text{inertial}}} \left[ B_{\text{earth},h} + \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N I}{r} \right]. \] (8)

Rewrite Equation (8) as

\[ \frac{1}{T^2} = C_0 + C_1 I, \] (9)

where:

\[ C_0 = \left( \frac{\mu}{4\pi^2 I_{\text{inertial}}} \right) B_{\text{earth},h}, \] (10)

\[ C_1 = \left( \frac{\mu}{4\pi^2 I_{\text{inertial}}} \right) \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N}{r}. \] (10*)

Measuring \(1/T^2\) as a function of \(I\) and fitting the measured \(1/T^2\)-versus-\(I\) curve by Equation (9), we obtain \(C_0\) and \(C_1\). Dividing Equation (10) by Equation (10*), we can derive \(B_{\text{earth},h}\) from

\[ B_{\text{earth},h} = \frac{C_0}{C_1} \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N}{r}. \] (11)

The SI units for the relevant physical quantities in the above equations are: \(B\) in tesla (T); \(r\) in meter (m); \(I\) in ampere (A); \(T\) in second (s). \(\mu_0 = 4\pi \times 10^{-7} T \cdot m / A\). The SI units for \(C_0\) is 1/s² and that for \(C_1\) is 1/(A·s²).

**EXPERIMENT**

**Apparatus**

The main part of the set-up used in this lab is the pair of Helmholtz coils shown in Figure 3. The radius of the coils is \(r = 0.105\ m\). Each coil is wound with \(N=130\) or \(125\) turns of wire. **Note: the maximum current for the Helmholtz coils is 1.0 A!!!** A small bar magnet is hanged approximately at the mid-way between the two coils by the hanging wire.

![Figure 3](image-url)
Procedures

1. Set up the Helmholtz coils and the bar magnet: Using the small bar magnet (when it stops rotating) to determine the direction of the local $B_{\text{earth,}h}$ (the horizontal component of the Earth’s magnetic field). Now, slowly reorient the Helmholtz coils to make the axis of the Helmholtz coils parallel to $B_{\text{earth,}h}$ (this may have been done by your TA). Slowly shift the hanging wire of the bar magnet to make the bar magnet align as horizontally as possible, to ensure that the bar magnet is hanged by the wire at its center-of-gravity. Check whether the bar magnet points along the axial axis.

Read the marked turns of wire wound in each coil and record $N$ in Table 1. The average radius of each coil is also given in Table 1.

2. Set up the circuit (Figure 4)

Connect the ground of the power supply to reference connector of the variable resistor. Connect the sliding connector to the common terminal of the ammeter. On the base of the Helmholtz coils, there are two connectors labeled as “Field”. Connect one “Field” connector to the positive terminal of the power supply, and the other to the positive terminal of the ammeter.

At this moment, the power supply should remain off. Ask your TA to check the circuit!

Note: the maximum current for the DC Power Supply is 2.0 A!!!

Now, turn on the power supply. Increase the current to 1.0 A. If the bar magnet rotates horizontally by 180º, it means that the Helmholtz coils produces $B_{\text{helm}}$ anti-parallel to $B_{\text{earth,}h}$, and you need to exchange the connections for the two “field” connectors. On the other hand, if they are connected as desired in this lab, the Helmholtz coils produces $B_{\text{helm}}$ parallel to $B_{\text{earth,}h}$.

3. Measure the time period $T$ when $I = 0$

Turn off the power supply. So $I=0$ and $B_{\text{helm}}=0$. Gently turn the bar magnet horizontally to a small angle ($<15^\circ$) off the axial axis (which is also the direction of $B_{\text{earth,}h}$). After being gently released, the bar magnet should undergoes simple harmonic motion by swinging back and forth around the axial axis under the restoring torque due to $B_{\text{earth,}h}$. Measure the time duration for 20 cycles of swing, $t_{20}$. Record the $t_{20}$ value in Table 2.

4. Measure the time period $T$ when $I \neq 0$

Turn on the power supply. Set the current $I$ in turn from 0.15 to 1.20 A in steps of 0.15 A. (Note: the maximum current for the DC Power Supply is 2.0 A!!!)

For each current setting, gently turn the bar magnet horizontally to a small angle ($<15^\circ$) off the axial axis. After being gently released, the bar magnet should undergoes simple harmonic motion. Measure the time duration for 20 cycles of swing, $t_{20}$. Record the $t_{20}$ value in Table 3.

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<tr>
<th>TABLE 1</th>
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TABLE 3

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Analysis

1. For the $t_{20}$ data in Table 2 and Table 3, calculate and record the corresponding time periods $T = t_{20} / v_0$ and also $I/T^2$.

2. Using the data in Table 3 (do not use the $I = 0$ data of Table 2), plot $I/T^2$ versus $I$, and use Equation (9) to fit the curve. Record the fitting parameters below

$C_0 = \ldots$ \hspace{1cm} $C_1 = \ldots$

3. Using Equation (11), the fitted $C_0$ and $C_1$, and the $r$ and $N$ values in Table 1, calculate $B_{\text{earth,}B}$ (Denote $B_{\text{earth,}B}^*$)

\[
B_{\text{earth,}B}^* = \frac{C_0}{C_1} \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N}{r} = \ldots
\]

4. Using Equation (10*), the fitted $C_1$, and the $r$ and $N$ value in Table 1, calculate

\[
C_2 = \frac{\mu}{4 \pi^2 I_{\text{inertial}}} = C_1 \left( \frac{5}{4} \right)^{3/2} \frac{r}{\mu_0 N}
\]

5. Using Equation (9), the calculated $C_2$ value, and the $I/T^2$ data measured at $I = 0$ (Table 2), calculate $B_{\text{earth,}B}$ (Denote as $B_{\text{earth,}B}^{**}$)

\[
B_{\text{earth,}B}^{**} = \frac{1}{C_2} \left( \frac{1}{I^2} \right) = \ldots
\]

Questions

1. Which value, $B_{\text{earth,}B}^*$ or $B_{\text{earth,}B}^{**}$, should be more accurate? Why?