1. What do you investigate in this lab?

2. Consider two parallel straight wires carrying electric current in opposite directions with the same magnitude of 2.0 A. The center-to-center distance between the two wires is 5.0 mm. One wire is infinitely long and the other wire is 0.30 m long. Is the magnetic force between the two wires repulsive or attractive? What is the magnitude of the magnetic force between them?
   (Answer: $F = 4.8 \times 10^{-5} \, N$)

3. What mass on the Earth surface will experience a gravitational force with the same magnitude as the above magnetic force?
   (Answer: $m = 4.90 \, mg$)
Magnetic Force and Current Balance

Name_______________________________ Lab Section__________

Objective

In this lab you will investigate magnetic force between two parallel current-carrying wires and will measure the fundamental constant $\mu_0$, the permeability in free space.

Background

(a) Magnetic force (also known as the Lorentz force) on a moving charge: When a charge $q$ moves with velocity $v$ in a magnetic field $B$, it experiences a magnetic force, $F$, which is perpendicular to both $v$ and $B$. If $v$ is perpendicular to $B$, the magnitude of the magnetic force is given by:

$$F = qvB.$$  \hspace{1cm} (1)

(b) Magnetic field generated by a current-carrying wire: The Figure on the right displays the magnetic field generated by an infinitely long straight wire carrying electric current of magnitude $I$. At a distance $r$ from the wire, the magnetic field generated by the current has a magnitude:
\[ B(r) = \frac{\mu_0 I}{2 \pi r}, \quad (2) \]

where \( \mu_0 \) is the permeability of the free space, \( \mu_0 = 4 \pi \times 10^{-7} \text{ T m/A} \).

The direction of the magnetic field is perpendicular to the direction of the current and obeys the right-hand rule, as depicted in the Figure.

**C) Magnetic force between two parallel current-carrying wires:**

Consider two parallel, straight, and long wires (1 & 2), separated by distance \( d \) and both carrying an electric current \( I \). As discussed in (a), the electric current in wire 1 generates a magnetic field on wire 2 which is perpendicular to wire 2 and has a magnitude of \( B(d) = \frac{\mu_0 I}{2 \pi d} \).

Because of the current in wire 2, the conduction charges in wire 2 move along the wire and have a drift velocity, which is perpendicular to the magnetic field generated by wire 1. Thus, the conduction electrons in wire 2 experience a magnetic force. The magnetic force induced by wire 1 on a segment of wire 2 of length \( L \) is the sum of the magnetic forces on all the conduction charges in the segment of wire 2. It is given by

\[ F = IL \frac{\mu_0 I}{2 \pi d} = \frac{\mu_0 L I^2}{2 \pi d}. \quad (3) \]

The magnetic force is attractive when the electric currents in the two wires run in the same direction; the magnetic force is repulsive when the electric currents in the two wires run in the opposite directions. The S.I. units for the relevant physical quantities in Equation (3) are: \( F \) in Newton (N), \( d \) and \( L \) in meter (m), \( I \) in Ampere (A).
EXPERIMENT

Apparatus

The main part of the set-up used in this lab is shown in Figure 1. Wire AB is a conducting rod and is fixed on the board of the current balance. The vertical position of wire AB is thus fixed. Wire CD is also a conducting rod and is part of the frame KCDK’. Except for a plastic rod connected between K and K’, the rest part of the frame is metallic and thus conducting. The frame has two supporting knife edges (not shown): one under K and the other under K’. The frame is supported only by the two knife edges which are placed on the two short parallel brass supporting beams on the board. It can thus rotate freely around the line connecting the two knife edges, allowing wire CD to move away from or towards wire AB.

First, adjust the knife edges until wire AB and wire CD directly face each other. As described in the Coulomb Balance, the frame is balanced mainly by the frame and weight P, and secondary by weight P’ (located under Rod KK’). By rotating it, weight P can move along the attached rod to change its lever arm. Weight P’, which can slide up and down (by rotating) to change its lever arm, is used to change the
time period of oscillation of the frame and to equilibrate the frame. Metallic pane Q is placed between two horseshoe magnets and is used to damp oscillation of the frame. Make sure that pane Q does not touch these magnets. Note: for small-angle rotation, the position of weight P’ is important for efficiently stopping oscillation of the frame and to increase the sensitivity of the experiment.

After adjusting P and P’, the frame should be able to rotate freely around the line connecting the two knife edges but its oscillation is efficiently damped; wire AB and wire CD are aligned parallel with each other, with wire CD above Wire AB, and separated by only a few millimeters.

Measure the distance \((d)\) between the central axes of rod AB and rod CD: To accurately measure the center-to-center distance, \(d\), between rod AB and rod CD you will use a laser, a mirror, and a scale. As shown in Figure 1, the mirror is fixed on a vertical frame attached to the horizontal rod KK’. Rod CD is perpendicular to the mirror and rod KK’ is parallel with both rod CD and the mirror. If \(D_1\) is the distance from the mid-point of rod CD to the mid-point between the two knife edges, when the frame rotates clockwise around the axis connecting the two knife edge by a small angle of \(\alpha\), the mid-point of rod CD (on its top surface) moves up by a distance of \(D_1 \sin(\alpha) \approx D_1 \alpha\) (Figure 3).
Direct the laser beam onto the mirror. Make sure that this incident beam is in the plane formed by the normal of the mirror and the midpoint between the two knife edges. Place the scale such that the reflected beam intercepts on the scale. Following the law of reflection, if the angle of incidence (between the incident beam and the normal of the mirror) is $\theta$, the angle of reflection (between the incident beam and the normal) is also $\theta$.

When the frame rotates clockwise around the axis connecting the two knife edges by angle $\alpha$, the mirror also rotates clockwise by angle $\alpha$. So does the normal of the mirror. Because the incident laser beam remains at the same direction, the angle of incidence is increased by $\alpha$ becoming $\theta + \alpha$. The angle of reflection then also becomes $\theta + \alpha$. Thus, the reflected beam changes direction by angle $2\alpha$ (Figure 4). In accordance, the intercept of the reflected beam on the scale moves up by a distance of $\Delta h = D_2 \sin(2\alpha) \approx D_2 \, 2\alpha$, where $D_2$ is the distance between the mirror and the scale.

To measure the distance between the top surface of rod AB and the lower surface of rod CD, $d'$, bring rod CD down to press against rod AB. Measure the intercept position of the reflected beam on the scale, $h_0$. This serves as the reference position. After releasing rod CD, the frame rotates around the axis connecting the two knife edges by angle $\alpha$, and rod CD moves up by distance $d' = D_1 \sin(\alpha) \approx D_1 \alpha$ and the intercept of the reflected beam on the scale also moves up to $h$ with $\Delta h = h - h_0 = D_2 \sin(2\alpha) \approx D_2 \, 2\alpha$. $\Delta h$ is large and can thus be easily measured, enabling accurate measurement of the distance between the two rods that is given by

$$d' = D_1 \alpha = D_1 \left(\frac{h-h_0}{2D_2}\right).$$

If the radius of the two rods is $r$, the distance between the centers of the two rods is given by

$$d = d' + 2r = D_1 \left(\frac{h-h_0}{2D_2}\right) + 2r.$$
Measure the magnetic force between the two rods: In this lab, the repulsive magnetic force \((F)\) between the two rods (when carrying electric current of a certain magnitude \(I\) in opposite directions) is measured indirectly by simultaneously placing a mass \(m\) on rod CD. If under these two forces the distance between the two rods remains unchanged, one can conclude that \(F\) equals \(mg\) in magnitude.

In this lab, first place a weight of mass \(m\) at the small pan on rod CD which brings rod CD down. Then, increase the current \(I\) in the two wires to increase the repulsive magnetic force between them. When the magnitude of the magnetic force reaches that of the weight, rod CD returns to its origin position. Meanwhile, the intercept position of the reflected beam on the scale is again \(h\). This condition is described by

\[
\mu_0 \frac{LI^2}{2\pi d} = mg.
\]

The permeability in free space can be determined by measuring the \(m\)-versus-\(I\) relation. The S.I. units to be used in Equation (6) are: \(I\) – the current \(A\); \(L\) – the length of rod CD in \(m\); \(d\) – the distance separating the plates in \(m\); \(m\) – the mass of the weight in \(kg\); \(g = 9.80\ m/s^2\).

**Procedures**

1. Measure the radii of rod CD and rod AB, and record their average radius \((r)\) in Table 1. Measure the length \((L)\) of rod CD. Measure the distance \((D_1)\) between the mid-point of rod CD and the mid-point of the two knife edges. Record these parameters in Table 1.

<table>
<thead>
<tr>
<th>(r) ((m))</th>
<th>(L) ((m))</th>
<th>(D_1) ((m))</th>
<th>(D_2) ((m))</th>
<th>(h_0) ((m))</th>
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2. Set up the circuit (Figure 2)

At this moment, the power supply should remain turned off. Connect end A of rod AB to the ground (black) of the power supply, connect end B of rod AB to end K of frame KCDK', connect end K of frame KCDK' to the ground of the ammeter, connect the positive terminal of the ammeter to one terminal of the rheostat, connect the other end of the rheostat to the positive terminal (red) of the power supply. Set the ammeter to the correct sensitivity (10 A). Ask your TA to check the circuit!

3. Align rod CD with rod AB

Place the two knife edges of the frame (as shown in Figure 1) on the two short parallel supporting brass beams on the board. Adjust the knife edges to make rod CD is directly above rod AB and parallel to each other. Note: the frame should stand only on the knife edges.

Make sure pane Q is between the two horseshoe magnets but not touch them. Adjust the position of weight P (mainly) and weight P' (secondary) to bring the distance between the two rods within a few millimeters. Make sure that the frame can freely rotate around the axis connecting the knife edges while oscillation slows down in a conveniently short period.

4. Set up the laser and the scale

Caution: Do not shine the laser beams (incident or reflected) on eyes. Turn on the laser and open its shutter. Adjust the laser set-up such that the plane formed by the laser, the scale, and the mirror is perpendicular to the axis connecting the two knife edges, and that the incident laser beam intercepts the mirror and the reflected beam intercepts the scale. Measure the distance \(D_2\) between the intercept spots of laser beams on the mirror and the scale. Record it in Table 1.
5. Measure the distance \( (d) \) between the central axes of rod AB and rod CD.

Read the intercept position \( (h) \) of the reflected beam on the scale. Record it in Table 1. Next, place the provided small coin in the pan on rod CD such that it moves down and presses against rod AB. Read the intercept position \( (h_0) \) of the reflected beam on the scale. Record it in Table 1.

<table>
<thead>
<tr>
<th><strong>Table 2</strong></th>
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<tr>
<td><strong>mass</strong></td>
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<td>10( \times ) 10^{-6}</td>
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<td>30( \times ) 10^{-6}</td>
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<td>40( \times ) 10^{-6}</td>
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6. Measure the current-versus-mass relation for the equal magnitude between the weight and the magnetic force between Rod CD and Rod AB.

Remove the coin. Place a mass \( m = 5 \text{ mg} \) in the pan on rod CD. Gradually increase the current \( I \) in the wires by reducing the resistance of the rheostat. When rod CD stops oscillating and the intercept position of the reflected beam on the scale again reaches \( h \). Record the values of \( m \) and \( I \) in Table 2.

Repeat the measurement for \( m = 10, 20, 30, 40 \text{ mg} \).

7. Repeat step 6 after reversing the current directions in Rod CD and Rod AB.

Reverse the connections to the power supply and to the Ammeter. Repeat step 6 and record the corresponding readings of the \( I \) values in Table 2.
**Analysis**

1. Use Equation (5) and the parameters in Table 1 to calculate the center-to-center distance between rod CD and rod AB.
   Record: \[ d = \]

2. For each mass, calculate the averaged \( l \) values from step 7 and step 8, then square the averaged \( l \) value. Record them in Table 2.

3. Plot \( m \)-versus-\((l_{average})^2\) and obtain the slope with linear fit.
   Record the slope =

4. According to Equation (6), the slope equals \( \frac{\mu_0 L}{2 \pi d g} \). With \( g = 9.80 \text{ m/s}^2 \), use the measured \( d \) and \( L \) value and the fitted slope to calculate the \( \mu_0 \) value.
   Record: \[ \mu_0 = \]

**Questions**

1. To use the measured \( d \), we assume the current flows along the central axes of rod CD and rod AB. Because of the repulsive forces, the conduction electrons in each rod however tend to move as far away from the other rod as possible. Considering this effect, should the actual \( \mu_0 \) value be higher or lower than the measured \( \mu_0 \) value? Why?
2. If the length of rod AB is doubled while the length of rod CD remains the same, will the result change?