Mid-Term Exam 1 - PHYS 300 - Modern Physics

Mendes, Spring 2010, Feb 09 2010

Start time: 11:00 a.m.
End time: 12:15 am

• Open textbook, notes, homeworks, and quizzes
• Calculators allowed; no other electronic device allowed
• Where it is appropriate, make sure to provide physical units to your numerical answer
• Make sure to provide your answer in the space indicated below each question
(20 points)

1.a) Compute the de Broglie wavelength of a proton of 4.5 keV kinetic energy.

\[ \lambda = \frac{\hbar}{p} \quad , \quad p = \left(2m_p k \right)^{\frac{1}{2}} = \left(2 \times 938.3 \times 10^6 \text{ eV} \times 4.5 \times 10^3 \text{ eV} \right)^{\frac{1}{2}} \]

\[ p = 2.91 \times 10^6 \text{ eV} \]

\[ \frac{p}{c} = 2.91 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \]

\[ \lambda = \frac{6.626 \times 10^{-34} \text{ Js}}{3 \times 10^8 \text{ m/s} \times 1.552 \times 10^{-21} \text{ J/s}} = 4.27 \times 10^{-13} \text{ m} \]

\[ \text{de Broglie wavelength} = \frac{4.27 \times 10^{-13} \text{ m}}{1.552 \times 10^{-21} \text{ J/s}} \]

1.b) Electrons in an electron microscope are accelerated from rest through an electric potential difference so that their de Broglie wavelength is 0.04 nm. What is the potential difference?

When \( V_0 \) is given in Volts, we have the following relation:

\[ \lambda = \frac{1.226 \text{ nm}}{V_0^{\frac{1}{2}}} \quad \Rightarrow \quad V_0 = \left(\frac{1.226 \text{ nm}}{0.04 \text{ nm}}\right)^2 \text{ V} \]

\[ V_0 = 939 \text{ V} \]

Potential difference = 939 V
(10 points)

2) Assume a 40 W sodium lamp (\( \lambda = 589 \text{ nm} \)) emitting light uniformly in all directions. Calculate the rate (number of photons / (unit area \( \times \) unit time)) at which the photons cross a surface placed normally to the beam at a distance 10 m from the light source.

\[
I = \frac{40 \text{ W}}{4\pi d^2} = \frac{40 \text{ W}}{4\pi \times (10 \text{ m})^2} = 0.03183 \frac{\text{W}}{\text{m}^2}
\]

\[
I = \frac{\eta \ h\nu}{h\ c} = \frac{I \ \lambda}{h\ c} = \frac{0.03183 \times 589 \times 10^{-3}}{6.626 \times 10^{-34} \times 3 \times 10^8}
\]

\[
\eta = \frac{0.943 \times 10^{17}}{\text{m}^2 \times \text{s}}
\]

\[
\text{number of photons} \quad \frac{\text{unit area} \times \text{unit time}}{\text{unit area} \times \text{unit time}} = \frac{0.943 \times 10^{17}}{\text{m}^2 \times \text{s}}
\]
(20 points)

3) The NaCl molecule has a bond energy of 4.26 eV; that is, this energy must be supplied in order to dissociate the molecule into neutral Na and Cl atoms.

a) What are the minimum frequency and maximum wavelength of the photon necessary to dissociate the molecule?

\[ h\nu = 4.26 \text{ eV} \Rightarrow \nu = \frac{4.26 \times 1.6 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J s}} = 1.03 \times 10^{15} \text{ Hz} \]

\[ c = \lambda \nu \Rightarrow \lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m/s}}{1.03 \times 10^{15} \text{ Hz}} = 2.91 \times 10^{-7} \text{ m} \]

\[ \text{frequency} = 1.03 \times 10^{15} \text{ Hz} \quad \text{wavelength} = 291 \text{ nm} \]

b) In what part of the electromagnetic spectrum is this photon?

\[ \text{Answer: UV} \]
(20 points)

4.a) Calculate at what wavelength the human body emits its maximum thermal electromagnetic radiation. Clearly indicate the assumed temperature for the human body.

Assume: \( T \equiv 310^\circ K = 37^\circ C = 98^\circ F \)

Wien’s Law:

\[
\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} \quad \Rightarrow \quad \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m}}{310 \text{ K}}
\]

\[
\lambda_{\text{max}} = 9.3 \times 10^{-6} \text{ m}
\]

wavelength = \( 9.3 \mu\text{m} \)

4.b) Assuming the human body to be an ideal blackbody, calculate the total thermal electromagnetic power per unit area radiated by the human body?

\[
R_t = \sigma T^4 = 5.67 \times 10^{-8} \frac{W}{\text{m}^2 \cdot \text{K}^4} \times (310 \text{ K})^4 = 524 \frac{W}{\text{m}^2}
\]

Total power per unit area = \( 524 \frac{W}{\text{m}^2} \)
(30 points)

5) X-rays of wavelength 0.24 nm are Compton-scattered, and the scattered beam is observed at an angle of 60° relative to the incident beam.

a) Find the wavelength of the scattered X-rays.

\[ \Delta \lambda = \lambda_e (1 - \cos \theta) \quad \text{for electrons} \quad \lambda_e = 0.00243 \text{ nm} \]

\[ \Delta \lambda = 0.00243 \text{ nm} \left(1 - \cos 60^\circ\right) = 0.001215 \text{ nm} \]

\[ \lambda_s = 0.24 \text{ nm} + 0.001215 \text{ nm} = 0.241215 \text{ nm} \]

wavelength = 0.241215 nm

b) Determine the energy of the scattered X-ray photons.

\[ h \nu_s = \frac{h \cdot c}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J s}}{0.24 \times 10^{-9} \text{ m}} \times 3 \times 10^9 \text{ m/s} = 8.241 \times 10^{-16} \text{ J} \]

\[ h \nu_s = 8.241 \times 10^{-16} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19}} = 5.151 \text{ keV} \]

Energy of scattered photons = 5.151 keV

c) Calculate the kinetic energy of the scattered electrons.

\[ h \nu_i = \frac{h \cdot c}{\lambda_i} = \frac{6.626 \times 10^{-34} \text{ J s}}{0.24 \times 10^{-9} \text{ m}} \times 3 \times 10^9 \text{ m/s} = 8.283 \times 10^{-16} \text{ J} \]

\[ h \nu_i = 5.177 \text{ keV} \quad \Rightarrow \quad K = 5.177 \text{ keV} - 5.150 \text{ keV} = 0.026 \text{ keV} \]

Energy of scattered electrons = 26 eV