Final Exam - PHYS 300 - Modern Physics
Mendes, Spring 2010, April 27 2010

Start time: 11:30 a.m.
End time: 2:00 am

- Open textbook, notes, homeworks, and quizzes
- Calculators allowed; no other electronic device allowed
- Where it is appropriate, make sure to provide physical units to your numerical answer
- Make sure to provide your answer in the space indicated below each question
1) A typical tungsten-filament lamp operates at about 3,000 °K with spectrum similar to a blackbody.

a) What is the wavelength of maximum emission of electromagnetic radiation?

\[
\text{Wavelength of strongest emission} = \underline{\text{____________}}
\]

b) What is the total power-per-unit-area emitted by this lamp over the whole electromagnetic spectrum?

\[
\frac{\text{power}}{\text{unit area}} = \underline{\text{____________}}
\]
(10 points)

2) Sketch the wave-function and the probability distribution corresponding to the quantum state $n = 4$ for a particle confined in a one-dimensional infinite square-well potential of length $= a$. 
(10 points)

3) A particle of mass of $10^{-6}$ g is moving freely with a constant speed of about $10^{-1}$ cm/s in a box of length 1 cm. Consider this problem as a one-dimensional infinite square-well potential.

a) Calculate the approximate value of the quantum number $n$.

$$n = \phantom{1234567890}$$

b) Sketch the wave-function and the probability distribution corresponding to this quantum state.
4) Consider a particular solution to the Schroedinger equation:
\[ \Psi(x,t) = A \cos \left( \frac{3\pi x}{a} \right) e^{\frac{-it}{\hbar}} \text{ for } \frac{-a}{2} < x < \frac{a}{2} \]
\[ \Psi(x,t) = 0 \text{ for } |x| > \frac{a}{2} \]
for a particle which moves freely through the region \( \frac{-a}{2} < x < \frac{a}{2} \) but which is strictly confined to it.

a) Calculate the expectation value of the linear momentum \( \overline{p} \).

\[ \overline{p} = \______________________________ \]

b) Calculate the expectation value of squared linear momentum \( \overline{p^2} \). You should get an answer that just depend on “a” and “\( \hbar \)”

\[ \overline{p^2} = \______________________________ \]
(10 points)

5) Determine the minimum angle that the angular momentum \( L \) can make with the z-axis when the angular momentum quantum is:

a) \( l = 2 \)

\[ \text{Minimum angle} = \frac{\pi}{2} \]

b) \( l = 4 \)

\[ \text{Minimum angle} = 0 \]
6) Positron is the anti-matter of the electron (same mass, but opposite electric charge). The electron-positron pair can form a hydrogen-like system called positronium. Keep in mind that in this case the mass of the nucleus is identical to the mass of the orbiting particle, and the approximation of an infinitely heavy nucleus is NOT valid and the concept of reduced mass is crucial.

a) Calculate the energy of the three lowest states of the positronium.

\[ \text{Energy state}_1 = \text{______________________________} \]

\[ \text{Energy state}_2 = \text{______________________________} \]

\[ \text{Energy state}_3 = \text{______________________________} \]

b) Determine the photon wavelength associated with a transition of the positronium system from \( n = 2 \) to \( n = 1 \).

\[ \text{Photon wavelength} = \text{______________________________} \]
(10 points)

7) A hydrogen atom in the ground state is placed in a uniform magnetic field of strength \( B_z = 1.00 \, \text{T} \).

a) Compute the energy splitting of the two spin states of the hydrogen atom in the ground state with respect to the z-direction of the magnetic field.

\[ \text{Energy splitting} = \quad \] 

b) If you wish to excite with electromagnetic radiation (photons) the hydrogen atom from the lower to the higher energy state calculated above, what wavelength must the photon have?

\[ \text{Photon wavelength} = \quad \]