HW #5

1. Plano-convex lens: \( R_1 = 50 \text{ mm}, \ R_2 = \infty, \ d = 3 \text{ mm}, \ \eta = 1.50 \)

\[
\frac{n_m}{f} = (\eta - n_m) \left[ \frac{1}{R_1} + \frac{(\eta - n_m) d}{R_2} \right] = (\eta - n_m) \left( \frac{1}{R_1} \right)
\]

\[
\frac{1}{f} = \frac{(1.50 - n_m)}{n_m} \Rightarrow f = \frac{n_m 50 \text{ mm}}{(1.50 - n_m)}
\]

Air: \( n_m = 1.00 \Rightarrow f = 100 \text{ mm} \)

Water: \( n_m = 1.33 \Rightarrow f = 391 \text{ mm} \)

2. \( f_1 = 50 \text{ mm} \)

\[
\frac{\phi}{2} = \frac{8(\phi/2)}{f_1} \Rightarrow \sqrt{f_2} = 8 f_1 = 8 \times 50 \text{ mm} = 400 \text{ mm}
\]

Separation: \( \overline{H_2 H_1} = f_1 + f_2 = 450 \text{ mm} \)

3. Diagrams of optical paths and lens configurations.
When \( L_2 \) is positioned at the object focal point of lens 1 (\( F_0 \)), the path of ray 0 does not change. It's ray 0 that determines the height of the image (it runs parallel to the optical axis). So, the new image will have the same height as the old image. In addition, note that, although the new image has the same original size, it is located at a different position: proper eyeglasses bring the image to the retina for those of us whose eyes are unable to do it.

9) \( R_0 = -2 \text{ m} \Rightarrow f_0 = \frac{-R_0}{2} = 1 \text{ m} \)

![Diagram showing the optical system with ray paths and focal points]

**PRIMARY:**

\[
\frac{1}{s_0} + \frac{1}{s_1} = \frac{1}{f_0} \Rightarrow s_{01} = f_0 = 1 \text{ m}
\]

**SECONDARY:** After reflection at the primary mirror, light propagates from right to left; one way to use the same sign convention (which was defined for light going from left to right) is to flip the secondary mirror and associated rays:

![Diagram showing the secondary mirror and ray path]

**IN THIS CASE:** \( s_0 = -0.25 \text{ m} \)

\( R_s = +0.6 \text{ m} \Rightarrow f_s = -\frac{R_s}{2} = -0.3 \text{ m} \)
$\frac{1}{s_{o2}} + \frac{1}{s_{z2}} = \frac{1}{f} \Rightarrow \frac{1}{s_{z2}} + \frac{1}{s_{z1}} = \frac{1}{f} \Rightarrow s_{z1} = 0.3 \times 0.25 \text{ m}$

$s_{z2} = 1.5 \text{ m}$

Refolding to the original configuration:

\[ f = 20.15 \text{ cm} \]
6. a) Double convex

\[ r = 50 \text{ cm} \]

Thickness: \( \frac{V_1}{V_2} = d = 5 \text{ cm} \)

\[ n = 1.5 \]

\[ \frac{1}{f} = (n-1) \left[ \frac{1}{r} - \frac{1}{-R} + \frac{d (n-1)}{n R (-R)} \right] = (n-1) \left[ \frac{2}{R} - \frac{d (n-1)}{n R^2} \right] \]

\[ \frac{1}{f} = (1.5-1) \left[ \frac{2}{50} - \frac{5 (1.5-1)}{1.5 \times 50^2} \right] \]

\[ f = 50.85 \text{ cm} \]

\[ \frac{V_1}{H_1} = -\frac{f d (n-1)}{n (-R)} = -50.85 \times 5 (1.5-1) \text{ cm} = 1.635 \text{ cm} \]

When combining 2 identical lenses separated by 20 cm (\( \overline{V_{12}} \overline{V_{21}} = \text{vcenter to vcenter} \))

The Distance between the Principal Planes \( \overline{H_{12}} \overline{H_{21}} \) (\( H_{12} = \text{second principal plane of first lens}, \overline{H_{21}} = \text{first principal plane of second lens} \)) is

\[ \overline{H_{12}} \overline{H_{21}} = 20 \text{ cm} + 2 \times 1.7 \text{ cm} \]

\[ S = 23.4 \text{ cm} \]

\[ \frac{1}{f_{\text{comb}}} = \frac{1}{f} + \frac{1}{f} - \frac{S}{f \cdot f} = \left( \frac{2}{50.85} - \frac{23.4}{50.85^2} \right) \]

\[ f_{\text{comb}} = 32.3 \text{ cm} \]

b) \( \overline{V_{12}} \overline{V_{21}} = 5 \text{ cm} \Rightarrow S = 8.4 \text{ cm} \)

\[ \frac{1}{f_{\text{comb}}} = \left( \frac{2}{50.85} - \frac{8.4}{50.85^2} \right) \]

\[ f_{\text{comb}} = 27.7 \text{ cm} \]
c) \( V_1 V_{k1} = 10 \text{ cm} \Rightarrow S = 13.4 \text{ cm} \)

\[
\frac{1}{\lambda_{comb}} = \left( \frac{2}{50.85} - \frac{13.4}{50.85^2} \right) \frac{1}{\text{cm}} \Rightarrow \lambda_{comb} = 28.3 \text{ cm}
\]

d) \( V_1 V_{k1} = 30 \text{ cm} \Rightarrow S = 33.4 \text{ cm} \)

\[
\frac{1}{\lambda_{comb}} = \left( \frac{2}{50.85} - \frac{33.4}{50.85^2} \right) \frac{1}{\text{cm}} \Rightarrow \lambda_{comb} = 37.9 \text{ cm}
\]

7. \( E_1(x,t) = E_0 \cos \left( (\kappa_0 + \Delta \kappa) x - (\omega_0 + \Delta \omega) t \right) \) \text{ same amplitude (} E_1 = E_0 = E_0 \text{)}

\( E_2(x,t) = E_0 \cos \left( (\kappa_0 - \Delta \kappa) x - (\omega_0 - \Delta \omega) t \right) \) \text{ same initial phase (} \phi_1 = \phi_2 = 0 \text{)}

\[
E(x,t) = E_1(x,t) + E_2(x,t) = E_0 \left\{ \cos A + \cos B \right\}, \quad A = (\kappa_0 + \Delta \kappa) x - (\omega_0 + \Delta \omega) t
\]

\[
B = (\kappa_0 - \Delta \kappa) x - (\omega_0 - \Delta \omega) t
\]

\[
\cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]

\[
E(x,t) = E_0 \left( \cos \left( \frac{\kappa_0 x - \omega_0 t}{2} \right) \cos \left( \frac{\Delta \kappa x - \Delta \omega t}{2} \right) \right)
\]

\( \kappa = \frac{\kappa_0 + \kappa_2}{2}, \quad \omega = \frac{\omega_0 + \omega_2}{2} \)

\( \Delta \kappa = \frac{\kappa_1 - \kappa_2}{2}, \quad \Delta \omega = \frac{\omega_1 - \omega_2}{2} \)

\[
\lambda_m = \frac{2\pi}{\Delta \kappa}, \quad \tau_m = \frac{2\pi}{\Delta \omega} \Rightarrow v_{m} = \frac{\lambda_m}{\tau_m} = \frac{\Delta \omega}{\Delta \kappa}
\]

wavelength of envelope, period of envelope
(6) \( \nu_{gn} = \frac{dw}{dk} \) (group velocity)

\[ \nu = \frac{w}{k} \] (phase velocity), \( \eta = \frac{C}{\nu} = \frac{C \cdot k}{w} \Rightarrow w = \frac{C \cdot k}{\eta} \]

\[ \nu_{gn} = \frac{dw}{dk} = \frac{c}{\eta} + c \frac{k}{\eta} \frac{d(\frac{1}{\nu})}{dk} = \frac{c}{\eta} - \frac{cK}{\eta^2} \frac{d\eta}{dk} \]

\[ k = \frac{2\pi}{\lambda} \Rightarrow k \cdot \lambda = 2\pi \Rightarrow \lambda \frac{dk}{d\lambda} + k \frac{d\lambda}{d\lambda} = 0 \Rightarrow \frac{dK}{d\lambda} = -\lambda \]

Then \( \nu_{gn} \frac{c}{\eta} - \frac{c}{\eta^2} \left( -\lambda \frac{d\eta}{d\lambda} \right) = \frac{c}{\eta} + \frac{c \cdot \lambda}{\eta^2} \frac{d\eta}{d\lambda} \)

(7) \( \Delta \lambda_0 = 1.2 \text{ nm}, \ \lambda_0 = 500 \text{ nm} \)

\( C = \lambda_0 \nu \Rightarrow 0 = \lambda_0 \frac{d\nu}{d\lambda} + \nu \frac{d\lambda_0}{d\lambda} \Rightarrow d\nu = -\frac{\nu}{\lambda_0} \frac{d\lambda_0}{d\lambda} \Rightarrow \)

\[ |\Delta \nu| = \frac{\nu}{\lambda_0} |\Delta \lambda_0| = \frac{c}{\lambda_0^2} |\Delta \lambda_0| = 3 \times 10^8 \times 1.2 \times 10^{-3} \text{ Hz} \approx 1.44 \times 10^{12} \text{ Hz} \]

\[ (500 \times 10^{-9})^2 \]

\[ C = \frac{C}{\Delta \nu} \frac{\lambda_0^2}{|\Delta \lambda_0|} = \frac{(500 \times 10^{-9})^2}{1.2 \times 10^{-3}} \text{ m} = 208 \mu\text{m} = 0.2 \text{ mm} \]