Mid-Term Exam 1 – PHYS 355 - OPTICS

Mendes, Fall 2008, Oct 02, 2008

Start time: 9:30 a.m.

End time: 10:45 am

Open textbook, notes, homeworks, and quizzes

Calculators allowed; no other electronic device allowed

Where it is appropriate, make sure to provide physical units to your numerical answer
1) The electric field of an electromagnetic plane wave propagating in a certain medium is given (in S.I. units) by:

\[ \vec{E}(x, y, z, t) = (5 \hat{e}_y + 7.5 \hat{e}_z) e^{i \left[ 6 \times 10^8 t - (\sqrt{3x + 3y^2 + 2z^2}) \times 10^8 \right]} \]

(5 points)

a) Determine the time period of a full cycle.

(5 points)
b) Calculate the phase velocity of this electromagnetic plane wave when propagating inside this medium AND when propagating in vacuum.

(5 points)
c) Calculate the refractive-index of the medium in which this electromagnetic wave is propagating.

(5 points)
d) Calculate the wavelength of the electromagnetic wave when propagating inside this medium AND when propagating in vacuum.

(5 points)
e) Find the equation for planes of constant phase.

(5 points)
f) Prove that the \( \vec{k} \)-vector and the electric field \( \vec{E} \) are perpendicular to each other.

(solve ONLY TWO out of g1, g2, and g3, 15 points each, 30 points total)

(2/7)
2) The plane $z = 0$ separates two media: glass ($n_{\text{glass}} = 1.65$ for $z < 0$) and water ($n_{\text{H}_2\text{O}} = 1.33$ for $z > 0$). The optical beam of a helium-cadmium (He-Cd) laser has a wavelength in vacuum of 442 nm. When this laser beam propagates in the x-z plane (as sketched below) from the glass side towards the glass/water interface at an angle of incidence of 30º (angle between the incident beam and the normal to the interface), determine the Cartesian components of the k-vector $\left( k_x, k_y, k_z \right)$ for the incident, reflected, and transmitted beams.
3) (10 points)

A material with a continuously varying refractive-index (also known as a graded-index material) can be modeled as a stratified stack of multiple parallel layers. Consider that 10 parallel layers of equal step in refractive-index are used to model a particular graded-index medium varying from 1.60 to 1.50, as shown below. For light incident from air, \( n_0 = 1.00 \), at an angle of incidence of \( \theta_0 = 30^\circ \), calculate the angle of refraction \( \theta_{10} \) that the light beam exits at the last interface of the stratified model.