Imaging Optics, Chapter 5 - Hecht

* Perfect image: point source is imaged to a point (image point)
  - image is a scaled version of an object

* Perfect image (impossible) ⇒ blur spot (good vs. bad image, limitations)

* Reversibility ⇔ conjugate points; interchange object & image

* Optical systems includes lenses, prisms, mirrors, etc

* Operation described by laws of reflection and refraction

* Wavefront ⇔ constant phase plane ⇔ plane perpendicular to $\mathbf{R}$ - vector plane propagates along $\mathbf{R}$

* Rays ⇔ power density propagation ⇔ Poynting vector ⇔ along $\mathbf{R}$

Lenses: redirects the propagation of an electromagnetic wave through refraction. Examples: collimation, focusing, magnification, etc

A Single, Curved, Refracting Surface:

\[
\frac{n_2}{n_1} = \frac{v_1}{v_2} \quad \text{slow} \\
\frac{n_2}{n_1} = \frac{v_2}{v_1} \quad \text{fast}
\]
How to transform a spherical wavefront into a flat wavefront?

\[
\frac{\overrightarrow{FA} + \overrightarrow{AD}}{v_i^*} = \text{constant} \Rightarrow \eta_s^* \frac{\overrightarrow{FA}}{v_i^*} + \eta_t^* \frac{\overrightarrow{AD}}{v_i^*} = \text{constant}
\]

\[
\eta_s^* s + \eta_t^* (x+t) = \eta_s^* \sqrt{(x+s)^2 + y^2} + \eta_t^* \frac{t}{s}
\]

\[
\frac{\eta_s^* s^* + \eta_t^* x^* + 2 \eta_s^* \eta_t^* s x}{s^*} = \eta_s^* \left( x^2 + 2sx + s^2 + y^2 \right)
\]

\[
(n_t^* - n_s^*) x^2 + 2 (n_t^* - n_s^*) s x - n_s^* y^2 = 0 \quad \text{(hyperbola)}
\]

\[
\text{positive} \quad \text{positive}
\]

\[
\eta_t^* \quad \eta_s^*\]

hyperbola \(\leftrightarrow\) flat wavefront in high refractive index medium
If $n_t < n_i \Rightarrow \text{ellipse}

\text{ellipse: flat wavefront in lower refractive index medium.}

\text{Aspherical Lenses}

\begin{align*}
\text{Plano - Convex} & \quad \text{Plano - Convex (Hyperbolic)} \\
\text{(Hyperbolic)} & \\
\text{Plano - Concave} & \quad \text{Double - Convex} \\
\text{(Hyperbolic)} & \\
\text{Plano - Convex} & \quad \text{Plano - Convex (Elliptical)}
\end{align*}

\text{Real and Virtual (Images and Object)}

\text{Virtual Image} \quad \rightarrow

\text{Virtual Object} \quad \rightarrow
**SINGLE SPHERICAL SURFACE:**

\[ S : \text{point source} \]
\[ P : \text{image point} \]
\[ C : \text{center of spherical surface} \]

\[ \overrightarrow{SP} = \overrightarrow{S_0} \]
\[ \overrightarrow{RP} = \overrightarrow{S_i} \]
\[ \overrightarrow{RC} = R \]

\[ \text{OPL} = n_1 l_0 + n_2 l_i \]

\[ l_0 = \left[ R^2 + (s_0 + R)^2 - 2R(s_0 + R) \cos \phi \right]^{1/2} \]
\[ l_i = \left[ R^2 + (s_i - R)^2 + 2R(s_i - R) \cos \phi \right]^{1/2} \]

\[ \frac{d(\text{OPL})}{d\phi} = 0 \Rightarrow n_1 \left[ \frac{2R(s_0 + R) \sin \phi}{-2l_0} \right] + n_2 \left[ \frac{-2R(s_i - R) \sin \phi}{-2l_i} \right] = 0 \]

\[ \frac{n_1(s_0 + R)}{l_0} = \frac{n_2(s_i - R)}{l_i} \Rightarrow \frac{n_1}{n_2} = \frac{l_0}{l_i} \frac{l_i}{l_0} = \frac{1}{R} \left( \frac{n_2 s_i - n_1 s_0}{l_i} \right) \]

**PARAXIAL RAYS**: \( \phi \approx 0^\circ \Rightarrow \cos \phi \approx 1 \Rightarrow s_0 \approx 0 \) and \( s_i \approx 0 \)

Then:
\[ \frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{1}{R} \left( \frac{n_2 - n_1}{s_i} \right) \]

*In the literature, several names are used to describe this branch of work: paraxial optics, first-order optics, and Gaussian optics all refer to the approximation above.*

*Paraxial properties (e.g., image position, image size) define the reference to measure deviation (aberration) when exact calculations are done.*
When $S_o \to \infty$ then $f_o \equiv S_o = \frac{n_k l}{(n_k-n_i)}$

When $S_o \to \infty$ then $f_i \equiv S_i = \frac{n_k l}{(n_k-n_i)}$

Sign convention: All positive in the figure below:

\[
\begin{align*}
S_i & = \text{Object} \\
S_o & = \text{Focal Point} \\
F_i & = \text{Image} \\
F_o & = \text{Focal Point}
\end{align*}
\]

\[
\begin{align*}
S_V &= S_o \\
F_o V &= f_o \\
V C &= l \\
S S_i &= \gamma_o \\
V F_i &= f_i \\
V P_i &= \gamma_i
\end{align*}
\]

Spherical lens:

\[
\begin{align*}
\frac{n_m + n_k}{S_{o,1}} &= \frac{1}{l_i} (n_k - n_m) \\
S V_i &= S_{o,1} \\
V_i P_i &= S_{i,1}
\end{align*}
\]

\[
\begin{align*}
V_i P_i &= S_{o,2} \\
S_{o,2} + S_{i,2} &= \frac{d - S_{i,1}}{V_i P_i}
\end{align*}
\]

\[
\begin{align*}
\frac{n_k + n_m}{S_{o,2}} &= \frac{1}{l_2} (n_m - n_k) \\
S_{i,2} &= S_{o,2} + S_{i,2}
\end{align*}
\]
\[ 0 + 2 : \eta_m \left( \frac{1}{s_{\theta,1}} + \frac{1}{s_{\lambda,2}} \right) + \eta_\ell \left( \frac{1}{s_{\lambda,1}} + \frac{1}{s_{\theta,2}} \right) = (\eta_\ell - \eta_m) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ \frac{s_{\theta,1} + s_{\lambda,1}}{s_{\lambda,1} s_{\theta,2}} = \frac{d}{s_{\lambda,1} (d - s_{\lambda,1})} \]

\[ \eta_m \left( \frac{1}{s_{\theta,1}} + \frac{1}{s_{\lambda,2}} \right) = (\eta_\ell - \eta_m) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) - \frac{\eta_\ell \frac{d}{s_{\lambda,1} (d - s_{\lambda,1})}}{s_{\lambda,1} (d - s_{\lambda,1})} \]
When \( S_{8,1} \to \infty \) then:

\[
\begin{align*}
\gamma_1 &= \gamma_0 \\
P &= F_2 \\
\sqrt{F_2} &= S_{2,2} \equiv b.f.l. \, (\text{back focal length}) \\
\overline{H_2 F_2} &= f \, (\text{effective focal length or efl})
\end{align*}
\]

\[
\begin{align*}
V_1 P_1 &= S_{i,1} = f_{i,1} = \frac{n_1}{n_2 - n_m} R_1 \\
\frac{n_m}{b.f.l.} &= (\frac{n_1 - n_m}{n_2 - n_m}) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) - \frac{n_2 d}{S_{i,1} (d - S_{i,1})}
\end{align*}
\]

\[
\begin{align*}
\frac{Y_2}{Y_1} &= \frac{(S_{i,1} - d)}{S_{i,1}} \\
\frac{Y_2}{b.f.l.} &= \frac{f}{f} \\
\frac{1}{S_{i,1}} &= \frac{1}{Y_1} \quad \frac{1}{b.f.l.} = \frac{(S_{i,1} - d)}{S_{i,1} \times b.f.l.}
\end{align*}
\]
\[
\frac{\eta_{m}}{f} = \frac{(s_{i,1} - d)}{s_{i,1}} \left[ (\eta_{e} - \eta_{m}) \left( \frac{1}{R_{i}} - \frac{1}{R_{z}} \right) - \frac{\eta_{e} \cdot d}{s_{i,1} \cdot (d - s_{i,1})} \right]
\]

\[
= (\eta_{e} - \eta_{m}) \left( \frac{1}{R_{i}} - \frac{1}{R_{z}} \right) \left[ 1 - \frac{d}{\eta_{e} R_{i}} \right] + \frac{\eta_{e} \cdot d}{s_{i,1}} \left( \frac{\eta_{e} - \eta_{m}}{\eta_{e} R_{i}} \right)^{2}
\]

\[
= (\eta_{e} - \eta_{m}) \left[ \left( \frac{1}{R_{i}} - \frac{1}{R_{z}} \right) - \left( \frac{1}{R_{i}} - \frac{1}{R_{z}} \right) \frac{d (\eta_{e} - \eta_{m})}{\eta_{e} R_{i}} + \frac{d (\eta_{e} - \eta_{m})}{\eta_{e} R_{z}} \right]
\]

\[
\frac{\eta_{m}}{f} = (\eta_{e} - \eta_{m}) \left[ \frac{1}{R_{i}} - \frac{1}{R_{z}} + \frac{d (\eta_{e} - \eta_{m})}{\eta_{e} R_{i}} \right], \quad f \equiv \frac{H_{2} F_{z}}{\eta_{e} R_{i}}
\]

\[
bfl = f \left( \frac{s_{i,1} - d}{s_{i,1}} \right) = f - f \cdot \frac{d (\eta_{e} - \eta_{m})}{\eta_{e} R_{i}}, \quad bfl \equiv \frac{V_{z} F_{z}}{\eta_{e} R_{i}}
\]

\[
\frac{H_{2} V_{z}}{\eta_{e} R_{i}} = f \frac{d (\eta_{e} - \eta_{m})}{\eta_{e} R_{i}}
\]
When \( S_{ij} \to \infty \) then:

\[
\begin{align*}
S &= F_0 \\
F_0 H_1 &= f' \\
\frac{\eta_m}{-f'} &= \left( \frac{\eta_2 - \eta_m}{\eta_2 - \eta_m} \right) \left[ \frac{1}{R_2} - \frac{1}{R_1} \right] + \left( -d \right) \frac{\left( \eta_2 - \eta_m \right)}{\eta_2 R_2 R_1} \\
\Rightarrow f' &= f
\end{align*}
\]

\[
\begin{align*}
F_0 V_i &= \text{ffl} \quad (\text{front focal length}) \\
-\text{ffl} &= -f - (-f) \left( -d \right) \frac{\left( \eta_2 - \eta_m \right)}{\eta_2 R_1} \\
\text{ffl} &= f + f d \frac{\left( \eta_2 - \eta_m \right)}{\eta_2 R_2}
\end{align*}
\]

\[
\begin{align*}
V_i H_i &= \frac{F_0 H_1 - F_0 V_i}{-f d \left( \frac{\eta_2 - \eta_m}{\eta_2 R_2} \right)} \\
&= \frac{-f d \left( \frac{\eta_2 - \eta_m}{\eta_2 R_2} \right)}{\eta_2 R_2}
\end{align*}
\]
**Principal Planes:**

\[ \frac{F_0}{R_1} = \frac{H_2}{F_{x}} = f ; \quad \eta_m = (n_k - n_m) \left[ \frac{1}{R_1} - \frac{1}{R_k} + \frac{d(n_k - n_m)}{\eta_k R_1 R_k} \right] \]

\[ F_0 \bar{V}_k = \bar{f} V_k = f + f \frac{d(n_k - n_m)}{\eta_k R_k} ; \quad \bar{V}_k H_k = -f \frac{d(n_k - n_m)}{\eta_k R_k} \]

\[ \bar{V}_i F_k = b s l = f - f \frac{d(n_k - n_m)}{\eta_k R_k} ; \quad \bar{H}_i V_k = f \frac{d(n_k - n_m)}{\eta_k R_k} \]

\[ f > 0: \quad F_0 \quad \bar{V}_k \quad \bar{H}_k \quad \bar{V}_k \quad \bar{F}_0 \]

\[ f < 0: \quad \bar{F}_0 \quad \bar{V}_k \quad \bar{H}_k \quad \bar{V}_k \quad \bar{F}_0 \]
The expression above, known as the Gaussian Lens Equation, is the fundamental relation for the imaging properties that will follow. You probably have seen a similar relation derived from the thin lens approximation. Notice that the equation above is more realistic in the sense it does not require the thin lens approximation. It's important that you notice that:

* the distances $s_o$ and $s_i$ are defined with respect to the principal points ($H_1$ and $H_2$, respectively).

* $f$ is given by  
  \[
  f = \frac{1}{R_1} + \frac{1}{R_2} + \frac{d}{R_1 R_2} (n_k - n_m)
  \]
**Transverse Magnification:** \( m_T = \frac{y_i}{y_o} \)

\[
m_T = \frac{y_i}{y_o} = \left( \frac{-s_i + f}{f} \right) = \frac{-s_i}{f} + 1 = \left( \frac{-s_i}{S_o} \right)
\]

From: \( \frac{y_o}{S_o} = -\frac{y_i}{S_i} \), we have:

An incoming ray that is aimed at \( H_1 \) will leave parallel from \( H_2 \).

**Focal length:** \( \frac{n_m}{f} = (\eta - n_m) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{d}{n_m} \frac{(\eta - n_m)}{n R_1 R_2} \right] \)

Typically \( n_m = 1.00 \) (air). But remember; if you immerse a lens in water, its focal length will not be the same as in air.

\[ 1.00 \left( \frac{n}{f} \right) 1.00 \Rightarrow \frac{1}{f} = (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{d}{n} \frac{(n - 1)}{n R_1 R_2} \right] \]
* focal length depends on refractive index, therefore if \( n = n(\lambda) \) then \( f = f(\lambda) \). This \( \lambda \)-dependence is the origin of chromatic aberration.

* a flat surface \( (R \to \infty) \) has no contribution to the focal length

* If \( R_1 = R_2 \):

\[
\frac{1}{f} = \frac{d(n-1)}{nR^2},
\]

* ball lens: \( \bigcirc \Rightarrow R_2 = -R_1 = -R \), \( d = 2R \)

\[
\frac{1}{f} = (n-1) \left[ \frac{2}{R} - \frac{2(n-1)}{R} \right] = (n-1) \frac{2}{R} \left[ 1 - \frac{(n-1)}{n} \right] = \frac{2}{R} \left( \frac{n-1}{n} \right)
\]

\[
V_1H_1 = -\frac{f}{nR_2} = \frac{Rn}{2(n-1)} \frac{2R(n-1)}{nR} = R
\]

* **Positive** (converging) lens: \( f > 0 \)

\[
\big[ \big[ \big[ \big] \big] : \text{center thicker than edge}
\]

(rule of thumb)

* **Negative** (diverging) lens: \( f < 0 \)

\[
\big( \big( \big( \big) \big) \big) : \text{edge thicker than center}
\]

(rule of thumb)
Image of real object: \[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

Positive lens: \( f > 0 \)

1. \( s_{o1} \rightarrow \infty \) than \( s_{i1} \rightarrow f \) \( (|m| = 0, \text{inverted, real}) \)
2. \( s_{o2} = 2f \) than \( s_{i2} = 2f \) \( (|m| = 1, \text{inverted, real}) \)
3. \( s_{o3} \rightarrow f \) than \( s_{i3} \rightarrow \infty \) \( (|m| \rightarrow \infty, \text{inverted, real}) \)
4. \( s_{o4} < f \) than \( s_{i4} < 0 \) \( (|m| > 1, \text{erect, virtual}) \)

Negative lens: \( f < 0 \)

\[ \frac{F_0}{H_1} = f < 0 \]
\[ \frac{H_2}{F_2} = f < 0 \]

\( (|m| < 1, \text{erect, virtual}) \)
Combination of lenses:

1. \( \frac{1}{H_2} + \frac{1}{F_2} = \frac{1}{H_1} \)
2. \( H_2 F_2 = f \)
3. \( \frac{-h'}{h} = \frac{d-f_1}{f_1} \)
4. \( s_{o,2} = d - f_1 \)

From 1 and 2:
\( \frac{1}{s_{o,2}} = \frac{1}{f_2} - \frac{1}{d-f_1} \)

From 3 and 4:
\( \frac{1}{-f} = \left( \frac{d-f_1}{f_1} \right) \frac{1}{s_{o,2}} \)

\( \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \)

From 1 + 2:
\( H_{22} F_2 = b \cdot f \cdot k \cdot s_{o,2} = \frac{f_2 (d-f_1)}{d-f_1-f_2} \)

\( H_{22} H_2 = H_{22} F_2 + F_2 H_2 = s_{o,2} - f = \frac{f_2 (d-f_1)}{d-f_1-f_2} - \frac{f_1 f_2}{f_1+f_2-d} \)
\[
H_{22} H_2 = \frac{f_2 d}{d - (f_1 + f_2)} = \frac{d}{f_1} = -\frac{f d}{f_1}
\]

Similarly, the front focal length can be determined:

\[
F_{o_1} H_{11} = \frac{f_1 f_1 f_2}{d - (f_1 + f_2)} = \frac{f_1 (d - f_2)}{d - (f_1 + f_2)}
\]

\[
H_{11} H_1 = +\frac{f d}{f_2}
\]
Mirrors:

Aspherical:

\[ A(x, y) \]

\[ S_v + \sqrt{p} = SA + AD \Rightarrow \]

\[ f + d = \sqrt{x^2 + (f-y)^2 + (d-y)} \Rightarrow y + f = \sqrt{x^2 + (f-y)^2} \Rightarrow \]

\[ y^2 + f^2 + 2fy = x^2 + f^2 + y^2 - 2fy \Rightarrow \]

\[ y = \frac{x^2}{4f} \]

Paraboloid: Collimates a point source.

Ellipsoidal and hyperboloidal also used for finite conjugates.

Spherical mirror:

\[ (y-R)^2 + x^2 = R^2 \Rightarrow \]

\[ y^2 + R^2 - 2yR + x^2 = R^2 \Rightarrow \]
\[ y^2 - 2yR + x^2 = 0 \Rightarrow \frac{y^2}{R^2} - \frac{2y}{R} + \frac{x^2}{R^2} = 0 \]

For \( \frac{y}{R} \ll 1 \Rightarrow \frac{2y}{R} = \frac{x^2}{R^2} \Rightarrow y = \frac{x^2}{2R} \)

So, in the paraxial approximation \( \left( \frac{y}{R} \ll 1 \right) \), \( 4f = 2R \Rightarrow \)

\[ f = \frac{R}{2} \]

Sign convention:

\[ \begin{array}{c}
\bullet & c \\
\cup & R < 0 \\
\cap & R > 0 \\
\cup & S_0 \quad S_i \\
\end{array} \]

So, left of \( \cup \) ; positive

\( \frac{1}{S_0} + \frac{1}{S_i} = \frac{1}{f} \)

\[ f = \frac{-R}{2} \]
Spherical Mirrors:

\[ \overline{SV} = s_o \]
\[ \overline{VP} = s_i \]
\[ \overline{VC} = R < 0 \text{ (concave)} \]

\[ \theta_i = \theta_o : \]
\[ \frac{\overline{SC}}{\overline{SA}} = \frac{\overline{CD}}{\overline{PA}} \]
\[ \overline{SC} = s_o - |R| \]
\[ \overline{CP} = |R| - s_i \]
\[ = s_o + R \]
\[ = -R - s_i \]

In the paraxial regime:
\[ \overline{SA} = s_o \]
\[ \overline{PA} = s_i \]

\[ \frac{s_o + R}{s_o} = \frac{-R - s_i}{s_i} \Rightarrow 1 + R \left( \frac{1}{s_o} + \frac{1}{s_i} \right) = -1 \Rightarrow \]

\[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{-2}{R} = \frac{1}{f} \]
Imaging with spherical mirrors:

a) Concave mirrors: $R < 0$, $f > 0$

- $|m| < 1$, inverted, real
- Same size ($|m| = 1$), inverted, real

- $|m| > 1$, inverted, real

b) Convex mirrors: $R > 0$, $f < 0$

- $|m| < 1$, erect, virtual
**Prisms:**
- Dispersive Element
- Beam Splitters
- Polarizing Devices
- Interferometers
- Image Orientation

**Example:** Angle of Minimum Deviation

\[ n = \frac{\sin \left( \frac{S \sin \gamma}{2} \right)}{\sin \left( \alpha/2 \right)} \]

High precision for refractive-index measurement!
1) Eyes:  - single-imaging lens
    - multiple tiny lenses coupled into channels
    - small holes

**Human Eye:**  - double, positive lens
    - Iris controls amount of collected light (pupil size)
      - 2-8 mm in diameter
    - Retina: thin layer (0.1-0.5 mm) of light receptor cells (rod & cones); concave light-sensitive screen
    - Rods: 100,10^6, 2 μm, black/white, high sensitivity
    - Cones: 6x10^5, 6 μm, color sensitive
    - Macula (3 mm): cones/rods = 2
    - Fovea (0.3 mm): just cones (1.0-1.5 μm)

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**Optical Instruments**

- **Cornea:** \( R_1 = 7.8 \text{ mm} \), \( t = 0.5 \text{ mm} \), \( n = 1.376 \)
- **Aqueous humor:** \( t = 3.0 \text{ mm} \), \( n = 1.336 \)
- **Eye lens:** \( R_3 = 10.1 \text{ mm} \), \( t = 4.0 \text{ mm} \), \( R_4 = -6.1 \text{ mm} \), \( n = 1.386-1.406 \)
- **Vitreous humor:** \( t = 16.3 \text{ mm} \), \( n = 1.337 \)

**f.f.l.** \( = 15.6 \text{ mm} \)

b.f.l. at retina

**Accommodation:** focusing done by the crystalline eye lens
2) **Eye glasses**

\[ D = \frac{1}{f} \text{ (diopters)} \] , "bending power"

\[ \frac{1}{f} = (\eta_k - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{d}{\eta_k R_1 R_2} \right] \]

Typically for eye glasses, \( d \approx \text{mm} \) and \( R \approx \text{meters} \), so in this case the thin-lens approximation is valid:

\[ D = \frac{1}{f} \approx (\eta_k - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \] (with length units in meters for units of diopters in \( D \))

**Far point:** longest distance the accommodated eye can image at the retina. Normal eye: longer than 5 m

**Near point:** shortest distance the eye can accommodate image at the retina. Normal eye: about 25 cm
Near-sightedness (myopia): image of distant object falls in front of retina

- Eye focal length is shorter than normal, too much bending power
- Far point is too close.
- Objects beyond far point appear blurred.
- "Closer than" "far" sharp.
- Add a negative (diverging) lens to bring object closer than the far point

\[
\frac{1}{f} = \frac{1}{f_\text{eyeglasses}} + \frac{1}{f_\text{eye}} \quad (s_\infty = \text{infinity}, \ s_i = \text{far point distance})
\]

\[
\frac{1}{f} = \frac{1}{D_\text{eyeglasses}} = \frac{1}{f_\text{eyeglasses}} \quad (\text{for point distance})
\]

Same magnification = same \( f \)

\[
\frac{1}{f} = \frac{1}{f_\text{eyeglasses}} + \frac{1}{f_\text{eye}} - \frac{1}{f_\text{eye}}
\]

\[
d = \frac{1}{f_\text{eye}} \implies \left| \frac{1}{f} \right| = \frac{1}{f_\text{eyeglasses}} + \frac{1}{f_\text{eye}} - \frac{1}{f_\text{eye}} = \frac{1}{f_\text{eye}}
\]

Combination of correction eyeglasses + eye has the same focal length as unaided eye, when \( d = \frac{1}{f_\text{eye}} \):
\[ d = \frac{1}{f_{\text{eye}}} \]

\[ d = \frac{1}{f_{\text{eye}}} \text{ when } \frac{H_{\text{og}}}{V_{\text{e}}} = \frac{1}{f_{\text{eye}}} = 15.6 \text{ mm} \]
Farsightedness: image of distant object falls behind the retina for the unaccommodated eye.

- Near point is longer than normal (≥ 25 cm)
- Any object closer than the near point cannot be imaged at the retina
- A positive (converging) lens is needed.

\[
\frac{1}{f_{eye}} = \frac{1}{S_0} + \frac{1}{S_i} - (\text{near point distance})
\]

\[
\text{Dyze} = 4 - \frac{1}{\text{near point distance}}
\]

Astigmatism: asymmetry in the cornea, different bending power at different axis, anamorphic
3) **Magnifiers**: create an image of a nearby object that is larger than the image seen by the unaided eye:

\[ \frac{y_i}{d_0} = \frac{\alpha_u}{d_0} = \frac{y_0}{d_0} \]

(Magnifying Power: \( MP \))

\[ MP = \frac{\alpha_u}{\alpha_w} = \frac{\frac{y_i}{L}}{\frac{y_0}{d_0}} = \frac{\frac{y_i}{d_0}}{\frac{y_0}{d_0}} = \frac{S_i}{S_o} \cdot \frac{d_o}{L} = \frac{S_i}{S_o} \cdot \left( 1 - \frac{S_i}{S_o} \right) \cdot \frac{d_o}{f} \cdot \frac{1}{L} \]

\[ \Rightarrow S_o = L - l \Rightarrow MP = \frac{d_o}{l} \left[ 1 + \frac{1}{f} \left( L - l \right) \right] \]

a) \( l = f \Rightarrow MP = \frac{d_o}{f} = d_o \cdot D \)

b) \( l = 0 \Rightarrow MP = d_o \left( \frac{1}{L} + \frac{1}{f} \right) \)

d) \( S_o = d_o \Rightarrow d_o = L \Rightarrow MP = 1 + \frac{d_o}{f} = 1 + d_o \cdot D \)

c) \( S_o = f \Rightarrow L = \infty \Rightarrow MP = \frac{d_o}{f} = \frac{d_o}{D} \)

**Example**: \( f = 0.1 \text{ m} \Rightarrow D = 10 \text{ diopters} \Rightarrow MP = \frac{0.25 \text{ m}}{0.1 \text{ m}} = 2.5 \times \)
4) **Eye piece**: collimates an intermediate image so the relaxed eye can image at the retina.

\[
f \quad \text{eye piece} \quad \text{eye} \quad MP = \frac{d_0}{D} = \frac{254 \text{ mm}}{f} \quad \text{Ex: } 10 \times \Rightarrow f = 25.4 \text{ mm}
\]

5) **Microscope**:

\[
MP = \left( \frac{f_{\text{obs}}}{f_{\text{obs}}} \right) \left( MP_{\text{eye piece}} \right)
\]

\[
\begin{align*}
\frac{f_{\text{obs}}}{f_{\text{obs}}} &= \frac{y_i}{y_o} = -\frac{S_i}{S_o} = \frac{f - S_i}{f_o} \\
\text{(as} \quad \frac{-S_i}{S_o} = 1 - \frac{S_i}{f_o} = \frac{f_0 - S_i}{f_o})
\end{align*}
\]

Most manufacturers use a standard \( S_i - f = 160 \text{ mm} \) (called tube length), so:

\[
MP = \frac{-160 \text{ mm}}{f_o} \left( \frac{254 \text{ mm}}{f_e} \right)
\]
**Numerical Aperture**: Amount of collected light by objective:

\[ N A = n_0 \sin \theta_m \]

Later, we'll see that the maximum optical resolution (imposed by diffraction effects) is proportional to \( \frac{\lambda}{N.A.} \).

6) **Telescopes**:
   a) **Refracting**

**Keplerian Telescope**:
- Two positive lenses separated by the sum of their focal lengths: \( d = f_o + f_e \)
- Inverted image

**Magnification**
\[
M_p = \frac{m_a}{m_u} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}
\]
Galilean Telescope: Positive OBJ + Negative Eyepiece

b) Reflecting:
   - Larger Aperture: More Light
   - Better Resolution
   - Easier w/ Mirrors than Lenses

   (No bulk material to cross, mechanical stability, chromatic aberrations)

Newtonian:

Gregorian:

P = paraboloid
E = ellipsoidal
H = hyperboloid
S = spherical

Cassegrain:

Ritchey-Chretien: Cassegrain Type with Both Primary & Secondary Hyperboloid to correct for spherical aberration and coma.

Ex: Hubble Space Telescope (Φ = 2.4 m, down to UV).
C) CATADIOPTRIC: REFLECTING AND REFRACTING ELEMENTS

SCHMIDT: - PRIMARY IS SPHERICAL TO INCREASE FIELD OF VIEW
- REFRACTING TO CORRECT SPHERICAL ABERRATION

[Diagram of a Schmidt-type telescope with labeled parts: COLLECTION PLATE, S]
Optical Aberrations

Aberration: deviation from ideal image (perfect scaled replica of object)

1) Monochromatic aberrations:

\[ \sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots \]

- Spherical
- Coma
- Astigmatism
- Field curvature
- Distortion

Spherical Aberration: focus depends on the ray height from axis.

\[ \frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h \left[ \frac{n_1}{2 s_o} \left( \frac{1}{s_0} + \frac{1}{R} \right)^2 + \frac{n_2}{2 s_i} \left( \frac{1}{R} - \frac{1}{s_i} \right)^2 \right] \]
* Orientation of lens affects SA:

\[ \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the short conjugate side}
\end{array} \rightarrow \text{better than} \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the long conjugate side}
\end{array} \rightarrow \]

* Shape of lens affects SA:

a) \[ \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the short conjugate side}
\end{array} \rightarrow \text{better than} \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the long conjugate side}
\end{array} \rightarrow \]

b) \[ \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the short conjugate side}
\end{array} \rightarrow \text{better than} \rightarrow \begin{array}{c}
\text{flat surface} \\
\text{on the long conjugate side}
\end{array} \rightarrow \]

* Sph. Aberration is determined for an on-axis object (it's known as on axis aberration)

**COMA:** a bundle of oblique rays will form images of different height (different magnification)

* Off-axis aberration*
* Shape and orientation affect coma

* Object at infinity, convex-planar lens is better choice (same that minimizes sph. aber.)

**ASTIGMATISM**:

Meridional plane: $OAA'$

Sagittal plane: $OBB'$

Optics by E. Hecht
FIELD CURVATURE: IMAGE PLANE IS NOT FLAT, BUT CURVED

\[ f = \frac{n_2 R}{n_2 - n_1} \]

DISTORTION:

Optics by E. Hecht
Mathematical Description:

\[ y' = A_1 \, s \, \cos \theta + A_2 \, h + \]
\[ B_1 \, s^3 \, \cos \theta + B_2 \, s^2 \, h \left( 2 + \cos 2 \theta \right) + \left( 3B_3 + B_4 \right) s \, h^2 \, \cos \theta + B_5 \, h^3 + \]
\[ C_1 \, s^5 \, \cos \theta + \cdots + C_{12} \, h^5 + \]
\[ \ldots \]

\[ z' = A_1 \, s \, \sin \theta + \]
\[ B_1 \, s^3 \, \sin \theta + B_2 \, s^2 \, h \, \sin 2 \theta + \left( B_3 + B_4 \right) s \, h^2 \, \sin \theta + \]
\[ C_1 \, s^5 \, \sin \theta + \cdots + C_{11} \, h^4 \, \sin \theta + \]
\[ \ldots \]

2) Chromatic aberration: dependence on wavelength \( \lambda \) through the refractive index \( n(\lambda) \).

\[ \frac{1}{f(\lambda)} = \frac{(n-1)}{R_1} - \frac{1}{R_2} + \frac{d(n-1)}{n \, R_1 \, R_2} \]