Mid-Term Exam - PHYS 355 - OPTICS

Mendes, Fall 2011, Oct 21

Start time: 10:00 a.m.

End time: 10:50 am

Open textbook, classnotes, and homeworks

Calculators allowed; laptops are restricted to instructor’s online material for this class.

Where it is appropriate, make sure to provide physical units to your numerical answer
1) A laser beam of frequency $6 \times 10^{14}$ Hz falls perpendicularly on a screen with two holes. One beam travels in glass, the other in air. How long will the glass need to be to cause a delay of 1 ns between the beam in glass relative to that in air? Take the refractive indices of air and glass to be 1.00 and 1.50, respectively.

\[
\nu_a = \frac{c}{1.00} = c, \quad \nu_g = \frac{c}{1.50} = \frac{2c}{3}
\]

Through glass: beam reaches distance $l$ at $t$ with: 
\[ l = t \nu_g \quad (1) \]

Through air: beam reaches distance $l'$ at $t - \Delta t$ with: 
\[ \Delta t = 1 \text{ ns} \quad \text{and} \quad l = \nu_a (t - \Delta t) \quad (2) \]

Combining (1) and (2) to eliminate $t$, we get:

\[
\frac{l}{\nu_g} = \frac{l'}{\nu_a} + \Delta t \Rightarrow l = \frac{\Delta t}{\nu_a - \frac{\nu_g}{\nu_a}} = \frac{\Delta t}{\frac{3}{2c} - \frac{1}{c}}
\]

\[ l = 2 \cdot 10^9 \text{ m} \cdot 10^{-9} \text{ s} = 2 \times 10^{-1} \text{ m} = 20 \text{ cm} \]
2) A lightwave travels from point A to point B in vacuum. Suppose we introduce into its path a flat glass plate \((n_g = 1.50)\) of thickness \(L = 1.00\) mm. If the vacuum wavelength is 500 nm, what is the phase difference for light propagating from A to B before and after the insertion of the glass plate?

\[
\phi_{A \to B} = \frac{2 \pi}{\lambda_o} n_o d_{AB}, \quad \text{with} \quad n_o \equiv 1 \quad (\text{vacuum})
\]

\[
\phi_w = \frac{2 \pi}{\lambda_o} n_A (d_{AB} - L) + \frac{2 \pi}{\lambda_o} n_g L
\]

So, we get:

\[
\Delta \phi = \phi_w - \phi_{A \to B} = \frac{2 \pi}{\lambda_o} n_A d_{AB} - \frac{2 \pi}{\lambda_o} n_o L + \frac{2 \pi}{\lambda_o} n_g L - \frac{2 \pi}{\lambda_o} n_o d_{AB}
\]

\[
\Delta \phi = \frac{2 \pi}{\lambda_o} (n_g - n_o) L = \frac{2 \pi}{500 \times 10^{-3} \text{ m}} (1.50 - 1.00) \times 10^{-3} \text{ m}
\]

\[
= 2 \pi \times 10^3 \text{ rad} \approx 6.28 \times 10^3 \text{ rad}
\]
3) Recent experiments have determined that the complex refractive index of silver is given by $(\bar{n} = 0.09 + 0.41i)$. Calculate the reflectance of light incident from air onto a flat silver surface at the normal angle of incidence (i.e., angle between the light beam and the normal to the surface is $0^\circ$).

\[
R = |r|^2 = \left| \frac{\eta_i - \eta_t}{\eta_i + \eta_t} \right|^2 = \left| \frac{1 - (0.09 + 0.41i)}{1 + (0.09 + 0.41i)} \right|^2
\]

\[
\approx \frac{(1 - 0.09)^2 + (0.41)^2}{(1 + 0.09)^2 + (0.41)^2} \approx 0.7346 = 73.46\%
\]
4) A beam of light in air strikes the surface of a smooth piece of plastic having an index of refraction of 1.55 at an angle with the normal of 20°. The incident light has component E-field amplitudes parallel and perpendicular to the plane of incidence of 10.0 V/m and 20.0 V/m, respectively.

(a) Determine the amplitude of the reflected and transmitted fields at each polarization.

(b) Determine the reflectance and transmittance at each polarization.

\[ \sin \theta_i = n_t \sin \theta_t \Rightarrow \sin \theta_t = \frac{1.00 \sin(20^\circ)}{1.55} \approx 0.22066 \]

\[ \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} \approx 0.97535 \]

\[ \cos \theta_i = \cos(20^\circ) \approx 0.93969 \]

\[ R_\perp = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{1 \times 0.93969 - 1.55 \times 0.97535}{1 \times 0.93969 + 1.55 \times 0.97535} = \]

\[ R_\perp \approx -0.2334 \quad , \quad |R_\perp|^2 \approx 0.0545 \]

\[ T_\perp = \frac{2 n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{2 \times 1.00 \times 0.93969}{1 \times 0.93969 + 1.55 \times 0.97535} \approx 0.76663 \]

\[ T_\perp = \frac{4 n_i \cos \theta_i n_t \cos \theta_t}{(n_i \cos \theta_i + n_t \cos \theta_t)^2} = \frac{4 \times 1 \times 0.93969 \times 1.55 \times 0.97535}{(1 \times 0.93969 + 1.55 \times 0.97535)^2} \]

\[ T_\perp \approx 0.9455 \]
\[ T_{il} = - \frac{\frac{t}{\cos \theta_i} - \frac{t}{\cos \theta_e}}{\frac{1}{0.93969} - \frac{1.55}{0.97535}} = - \frac{1.00}{0.93969} + \frac{1.55}{0.97535} \approx + 0.19786 \]

\[ R_u = |T_u|^2 = 0.0392 \]

\[ t_{il} = \frac{\cos \theta_i}{\cos \theta_e} \left( \frac{\frac{t}{\cos \theta_i} + \frac{t}{\cos \theta_e}}{0.97535} \right)^2 = \frac{1.00}{0.93969} \left( \frac{1}{0.93969} + \frac{1.55}{0.97535} \right) \]

\[ t_{il} \approx 0.77281 \]

\[ T_{il} = \frac{4 \times \frac{t}{\cos \theta_i} \frac{t}{\cos \theta_e}}{\left( \frac{\frac{t}{\cos \theta_i} + \frac{t}{\cos \theta_e}}{0.93969} \right)^2} = \frac{4 \times 1.55}{0.93969} \frac{0.97535}{0.97535} \]

\[ T_{il} \approx 0.9609 \]

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Now, we can address the questions: \( E_{i,il} = 10 \frac{V}{m} ; E_{i,1} = 20 \frac{V}{m} \)

a) \( E_{T,il} = T_{il} E_{i,il} = + 0.19786 \times 10 \frac{V}{m} = + 1.9786 \frac{V}{m} \)
\[ E_{t,\|} = t_\| E_{z,\|} = 0.77281 \times 10 \frac{V}{m} = 7.7281 \frac{V}{m} \]

\[ E_{t,\perp} = t_\perp E_{z,\perp} = -0.2334 \times 20 \frac{V}{m} = -4.668 \frac{V}{m} \]

\[ E_{t,\perp} = t_\perp E_{z,\perp} = 0.76663 \times 20 \frac{V}{m} = 15.3326 \frac{V}{m} \]

b) \( R_\perp \approx 0.0545 \); \( T_\perp \approx 0.9455 \)

check: \( R_\perp + T_\perp = 1 \)

\( R_\| \approx 0.0392 \); \( T_\| \approx 0.9609 \)

check: \( R_\| + T_\| = 1.0000 \)

round-off error