1. The light emitted from a Nd:YAG laser at a wavelength of 1.06 μm is a Gaussian beam of 1-W optical power and beam divergence of \(2\theta_o = 1\, \text{mrad}\). Determine the following parameters:

a) The beam waist radius.

b) The depth of focus.

c) The maximum beam intensity.

d) The intensity on the beam axis at a distance \(z = 100\, \text{cm}\) from the beam waist.

\[
\begin{align*}
a) \quad W_o &= \frac{\lambda}{\pi \theta_o} = \frac{1.06 \times 10^{-6}}{\pi \times \frac{1}{2} \times 10^{-3}} = 0.675 \times 10^{-3} \, \text{m} = 0.675 \, \text{mm} \\

b) \quad z_o &= \frac{W_o}{\theta_o} = \frac{0.675 \times 10^{-3}}{\frac{1}{2} \times 10^{-3}} = 1.350 \, \text{m} \\

c) \quad I_o &= \frac{2P}{\pi W_o^2} = \frac{2 \times 1 \, \text{W}}{\pi (0.675 \times 10^{-3} \, \text{m})^2} = 1.399 \times 10^6 \, \text{W/m}^2 \\

d) \quad z &= 1 \, \text{m} \quad , \quad f = 0 \\
\quad I &= I_o \left(\frac{W_o}{W(2)}\right)^2 e^{-\frac{2z^2}{W^2(2)}} = I_o \cdot \frac{1}{\left(1 + \left(\frac{z}{z_o}\right)^2\right)^\frac{1}{2}} = \frac{1.399 \times 10^6 \, \text{W/m}^2}{\left[1 + \left(\frac{1 \, \text{m}}{1.350 \, \text{m}}\right)^2\right]} \\
\quad I &= 0.903 \times 10^6 \, \text{W/m}^2
\end{align*}
\]
2) The transmittance of a symmetric Fabry-Perot resonator was measured by using light from a tunable monochromatic light source. The transmittance versus frequency exhibits periodic peaks of period 150 MHz, each of width (FWHM) 5 MHz. Assuming that the medium within the resonator mirrors is a gas with \( n = 1.00 \), determine the following:

a) The length of the resonator.

\[
d = \frac{c}{2 \nu_F} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 150 \times 10^6 \text{ Hz}} = 1 \text{ m}
\]

b) The finesse of the resonator.

\[
f = \frac{\nu_F}{\delta \nu} = \frac{150 \times 10^6 \text{ Hz}}{5 \times 10^6 \text{ Hz}} = 30
\]

c) Assuming further that the only source of loss in the resonator is associated with mirrors, find their reflectance.

\[
f \gg 1 \quad \Rightarrow \quad \frac{\pi}{\alpha_r d} = \frac{\ln \left( \frac{1}{R^2} \right)}{\alpha_r d} = -\frac{\ln R}{\alpha_r d}
\]

\[
R = e^{-\alpha_r d} = e^{-\frac{\pi}{f}} = e^{-\frac{\pi}{30}} \approx 0.501 = 90.1\%
\]
3) The optical transition of the ruby laser Cr$^{3+}$:Al$2$O$_3$ has (to a good approximation) a Lorentzian shape of width (FWHM) 330 GHz centered at about 694 nm (vacuum wavelength). The measured peak of this transition cross section is $2.5 \times 10^{-20}$ cm$^2$. Consider that the refractive index of ruby is 1.76.

a) Calculate the radiative lifetime of this transition.

b) Consider that the observed lifetime for this transition, which includes radiative and non-radiative decay rates, is 3 ms. Calculate which fraction of the atoms in the upper state undergoes radiative transition and which fraction undergoes non-radiative transition.

\[
\tau_{\text{ap}} = \frac{\left(\frac{\lambda}{n}\right)^2}{8\pi \tau_{\text{ap}} n \Delta \nu} = \frac{4\pi^2 \tau_{\text{ap}} \Delta \nu}{\frac{4}{\pi^2} \left(6.34 \times 10^{-3} \text{ m} / 1.76\right)^2} \approx 4.8 \text{ ms}
\]

\[
\tau_{\text{ap}} = \frac{4}{\pi^2} \left(6.34 \times 10^{-3} \text{ m} / 1.76\right)^2 = 4.8 \text{ ms}
\]

\[
\frac{1}{\tau_{21}} = \frac{1}{\tau_{\text{ap}}} + \frac{1}{\tau_{\text{nr}}} = \frac{1}{3 \text{ ms}}
\]

\[
\frac{1}{\tau_{\text{ap}}} = \frac{1}{4.8 \text{ ms}}
\]

\[
\frac{1}{\tau_{\text{ap}}} = \frac{3 \text{ ms}}{4.8 \text{ ms}}
\]

\[
\text{Radiative: } \frac{1/\tau_{\text{ap}}}{1/\tau_{21}} = \frac{\tau_{21}}{\tau_{\text{ap}}} = \frac{3 \text{ ms}}{4.8 \text{ ms}} = 0.625 = 62.5\%
\]

\[
\frac{1}{\tau_{\text{ap}}} = \frac{1}{\tau_{21}} - \frac{1}{\tau_{\text{nr}}} = 1 - \frac{\tau_{21}}{\tau_{\text{ap}}} = 0.375 = 37.5\%
\]

\[
\text{Non-radiative: } \frac{1/\tau_{\text{nr}}}{1/\tau_{21}} = \frac{\tau_{\text{nr}}}{\tau_{21}} = \frac{1}{\tau_{\text{ap}}}
\]

\[
\frac{1}{\tau_{\text{ap}}} = \frac{1}{\tau_{21}} - \frac{1}{\tau_{\text{nr}}} = 1 - \frac{\tau_{21}}{\tau_{\text{ap}}} = 0.375 = 37.5\%
\]