Outline for Chapter 16: Evolution of Stars
Univ. Louisville, 14 Nov 2007

HW12, due Dec. 3:
16.9 (make table as on class website)
19.3, 19.10, 19.13 (100 word max, shorter is better)
21.2, 21.8 (100 word max), 21.9 (use mass(Gal)=3.4e11 $M_\odot$, 4=18-10 kpc)

– stick to normal stars (no time to do peculiar ones, sadly)

16-1A: Physical Laws of Stellar Structure: Hydrostatic Equilibrium

recall hydrostatic equilibrium:
$dP/dr = -GM(r)\rho(r)/r^2$ (Eqn 16-1)
where
$dM/dr = 4\pi r^2\rho(r)$ (Eqn 16-2) – mass continuity, with
total mass $M = 4\pi \int_0^R \rho(r)r^2 dr$ (Eqn 16-3)

so, need $\rho(r) -> M(r) -> P(r)$

Sun's Central Pressure

Plugging in solar values $R_\odot$, $M_\odot$, $\langle\rho_\odot\rangle = 1410$ kg/m^3 to Eqn 16-1 gives
$P_0 \approx 10^{14}$ N/m^2 ($\sim 10^9$ atm)
Differentiation (dense core) means actual pressure is higher

16-1B Equations of State

perfect gas law: $P(r) = n(r)kT(r)$ (Eqn 16-4)
remember reduced mass: $n(r) = \rho(r)/\mu(r)m_H$ (Eqn 16-5)
depends on mean molec weight $\mu = [2X + (3/4)Y + (1/2)Z]^{-1} \approx 0.5$ (Eqn 16-6)
Thus
$P(r) = \rho(r)kT(r)/(\mu(r)m_H)$ (Eqn 16-7)
* only thing we need is $\rho(r)$ because $\mu(r)$ is usually fixed for a star

For hot, luminous (massive) stars, also need radiation pressure:
$P_{rad} = (a/3)T^4(r)$, $a = 7.6 \times 10^{-16}$ J m^{-3} radiation constant

Sun's Central Temperature

fm perfect gas law, $T_c \approx P_c\mu m_H/\langle\rho_\odot\rangle k$ (Eqn 16-8)
For $\mu = 0.5$ get $T_c \approx 1.2 \times 10^7$ K (only 20% below numerical models) – of course totally ionized

16-1C Modes of Energy Transport

heat flow from hot core to photosphere:
1) Conduction (most efficient in solids/metals)
2) Convection (mass flow) - in fluids; most stars have convective zone

3) Radiative transport - photons scatter/absorb in random walk out
– opacity sources:
—- (free) electron scattering
—- photoionization
* all the complaints you hear about radiative transport are probably true **

derive eqn of radiative transport; safely assume Sun's interior = shells of blackbodies
flux $F(r) = \sigma T^4(r)$ (σ = Stefan-Boltzmann ct fm Chap 8)
$dF = 4\sigma T^3(r)$ differential (Eqn 16-10a)
From opacity (Chap 10): $dF = -\kappa(r)\rho(r)F(r)dr$ (Eqn 16-10b; κ=opacity in m^2/kg)
define luminosity $L = 4\pi r^2 F(r)$
equate dF expressions, substitute in luminosity expression to get
$L(r) = [(-16\pi\sigma r^2 T^3(r))/(\kappa(r)\rho(r)][dT/dr]$ (Eqn 16-11)

Need another factor of 4/3 to multiply above by. The factor of 4/3 simply comes from the fact that the radiation is spherically isotropic so that you need to average the emission angle. E.g. radiation in the direction perpendicular to the radius of the star does not contribute to the transfer at all and the contribution increases as the inclination to the radius decreases. (from Chris Tout, IOA, 2006)

If opacity is sufficiently high, convection more efficient than radiative transfer. *Opacity gets lower with higher temperature.*

Define $\gamma = c_p/c_v = 5/3$ = ratio of specific heats at ct pressure and volume, for fully ionized, ideal gas - this is *equation of state*
then for *convective transport*
$dT/dr = (1 - 1/\gamma)[T(r)/P(r)]dP/dr$ (Eqn 16-12b)

Whichever eqn is used (radiative or convective) depends on which is more efficient for particular star

Solar Luminosity fm Radiative Transfer

For Sun, most of interior uses *radiative transport*
estimate $dT/dr = -T_c/R_\odot \sim -0.02$ K/m. (So goes 2K/km - like Earth but 100x farther!)
Use R, T, $\langle\rho\rangle$ for Sun, use extra factor of 4/3 in radiative eqn, gives
$L_\odot \approx (9.5 \times 10^{29}/\kappa)$ Watts
Need to calculate opacity κ: for fully ionized H, have 6×10^{26} protons, same no. e-'s
$\pi a_e^2 \sim 10^{-30}$ m^2 (electron scattering)
$\pi a_p^2 \sim 10^{-20}$ m^2 (photo ionization)

In solar interior, photo ionization dominates over e- scattering

So $10^{-3} \ll \kappa \leq 10^7$, (corresponding to $10^{-10} - 10^1$ kg/m^3 for pure photonionization)
so $10^{22} \leq L_\odot \ll 10^{32}$ Watts; midpoint of $L = 10^{27}$ Watts is close to measured value 3.90×10^{26} Watts $->$ $\kappa \sim 2.4 \times 10^3$

THESE ARE ROUGH CALCULATIONS AND IT'S IMPRESSIVE HOW CLOSE THEY ARE TO DETAILED MODELS!

16-1D Energy Sources

stars are *not* static - they radiate lots of energy!
need heat sources: radiate energy $\epsilon(r)$ (W kg^{-1}, fn of T,ρ)
$\epsilon_\odot \approx L_\odot/M_\odot = 2.0 \times 10^{-4}$ W kg^{-1}

then must add luminosity term
$dL = 4\pi r^2 \rho(r)\epsilon(r)dr$
to luminosity equation wherever energy is generated

Gravitational Contraction

acc. to virial theorem, gravitational potential of contracting body decreases TWICE as fast as internal heat increases – so rest RADIATES AWAY, e.g.
$U = KE + PE = -2E_{therm}$

Gravitational potential is
$dU = -GM(r)dM(r)/r$, so integrating gives
$U = -\int_0^M GM(r)dM(r)/r = -q(GM^2/R)$
where q depends on mass distribution;
$q = 3/5$ for uniform sphere, $q \approx 1.5$ for most main-seq stars

energy PER KG of Sun available for gravitational radiation is
$GM_\odot/2R_\odot = 9.54 \times 10^{10}$ J/kg
So using $\epsilon_\odot = 2.0 \times 10^{-4}$ W kg^{-1} $->$
lifetime $t = 9.54 \times 10^{10}/2.0 \times 10^{-4}$ s
or 15 Myr – but we know Earth is 4 Gyr old from radioactive dating!

Thermonuclear Reactions

Fusion can provide energy for $L_\odot$.

Protons have ~ 1 MeV barrier to overcome, and $E_p \sim 1$ keV at 10^7 K
BUT, qtm mech allows tunnelling through potential to allow p-p reaction at 10^7 K

If m_H=1.0078 amu, m_{He} = 4.0026 amu, then *mass defect* from 4 H atoms to one ^{4}He is $\Delta m = 0.0286$ amu, so energy to make ^{4}He from H is $E = 4.3 \times 10^{-12}$ J,

so mass fraction $\Delta m / 4m_H = 0.0071$ is used for fusion

If 10% of Sun (core) available for fusion, total E in Sun is
$E_{tot} = (\Delta m)c^2(0.1 M_\odot)/[4m(^1H)]$
$\approx 10^{44}$ J

then $E_{tot}/L_\odot \approx 10$ Gyr. That's more like it!

Two fusion processes turn H to He:
PP chain (dominates $T < 2 \times 10^7$ K, dominates in Sun)
CNO cycle (dominates $T > 2 \times 10^7$ K, dominates in A-stars and earlier)

main PP chain: PP I, occurs 91% of time in Sun
$H + H -> D + e^+ + \nu$ (1.44 MeV)
$D + H -> {}^3He + \gamma$ (5.49 MeV; D=deuterium)
${}^3He + {}^3He -> {}^4He + H + H$ (12.9 MeV)
SIX protons are used in process; TWO are returned as H
ν carry off 0.26 MeV per reaction; now you know why we look for ν!

temperature dependence for PP I rate is $\propto T^4$

There are PP II and PP III chains which go through Li and Be as intermediaries and more ν.

Since 1965, we thought we only detected 1/3 to 1/4 of ν predicted from Sun. ν were assumed massless, and only electron-type ν_e (there are also tau ν_τ and muon ν_μ).

Sun predicted to produce only ν_e. Experiments till late 1990s mostly detected only ν_e.

Turns out $\nu_e -> \nu_\tau$ and ν_μ on way to Earth, via ν-oscillations - which means they must have mass!

See story on links site (chapter on Sun).

CNO Cycle - uses ^{12}C as catalyst to turn H into He, requires higher temperature

temperature dependence for PP I energy generation rate is $\propto T^4$
temperature dependence for CNO energy generation rate is $\propto T^{20}$
crossover at $T \sim 1.8 \times 10^7$ K

HIGHER TEMPERATURES:
triple-alpha process: 3 $^{4}He -> {}^{12}C$, using Be as intermediary
D, Li, Be, B rare in stellar interiors because they combine with H, make several He at $T \sim$ few million K
This is why D is formed only in Big Bang and never made in stars!

There are other He-burning reactions, too, at higher T
sequence of burning: C, Ne, O, Mg, etc. up to Fe
Fusing to make anything heavier than Fe is ENDOTHERMIC!

16-2 Theoretical Stellar Models: A. Recapitulating the Physics

Stellar model variables:
T(r), M(r), $\rho(r)$, P(r), $\epsilon(r)$ (power/unit mass), $\mu(r)$ (chem comp), κ (opacity - don't forget!)

Basic Eqns of stellar structure - lots-o-differential equations:
Hydrostatic Eq'm: $dP/dr = -GM(r)\rho(r)/r^2$
Mass Continuity: $dM/dr = 4\pi r^2\rho(r)$
Energy Transport - Radiative: $dT/dr = [(-3\kappa(r)\rho(r)/64\pi\sigma r^2 T^3(r)]L(r)$
Energy Transport - Convective: $dT/dr = (1-1/\gamma)[T(r)/P(r)]dP/dr$
Energy Generation (Thermal Eq'm): $dL/dr = 4\pi r^2\rho(r)\epsilon(r)$
Equation of State: $P(r) = k\rho(r)T(R)/[\mu(r)m_H]$

Boundary Conditions:
At ctr (r=0):
M(r)=0
L(r)=0

At "surface":
M(r)=M
L(r)=L
$T(r) = T_{eff}$
$\rho(r) -> 0$
$P(r) -> 0$

Also need:
$\epsilon(r)$ varies between nuclear/gravitational
$\kappa(\rho, T)$ depends on chem comp X,Y,Z (or equivalently μ)
$\mu(r)$ - which influences $\rho(r)$, $P(r)$ sizes of radiative, convective zones
– in Sun, core is He-enriched (60%), and convective zone is 0.7-1.0$R_\odot$ (needs steep dT/dr for convection - see by granules)

Equations are solved numerically, though some analytic approximations are useful.

16-2B Physical Basis of M-L Relation

Differential equations can look daunting. We learn relations between variables by making some rough estimates. Start with hydrostatic eq'm. Instead of dP/dr, take $\Delta P/\Delta r$ evaluated between core and surface:
$P_c \approx GM\langle\rho\rangle/R \propto M\langle\rho\rangle/R$
For a perfect gas, $P \propto \rho T$
Thus
$\rho T \propto M\rho/R$
or at the core
$T_c \propto M/R$
Do same with radiation transport:
$dT/dr = [-3\kappa\rho/64\pi\sigma r^2 T^3]L$
so
$T_c/R \propto \kappa\rho R^{-2} T_c^{-3} L$
or $L \propto RT_c^4/\kappa\rho$

put in $\rho \propto M/R^3$ to get
$L \propto R^4 T_c^4/(\kappa M)$

now substitute in relation from hydrostatic eq'm $T_c \propto M/R$
$L \propto R^4(M/R)^4/(\kappa M)$
$L \propto M^3/\kappa$
Remember that observed relation is $L \propto M^\alpha$, $\alpha \approx 2.3, 4.0$ ($M < 0.43M_\odot, > 0.43M_\odot$).

16-3 Stellar Evolution
general steps: proto-star, pre-main seq, main seq, post-main seq
Evolution is governed by mass! (and also by chem composition; also winds, companions if there is mass transfer)
Evolutionary Track: ≡movement on H-R diagram

16-3A Star Births: Protostars & PMS Stars

As last chapter:
collapse of dense ISM cloud converts grav'l energy to 50% thermal, 50% radiative energy
$->$ core heats, fusion starts, stops being proto-star when gets hydrostatic eq'm
Before fusion starts, is pre-main-sequence (PMS) star, is on PMS evol'nary track
PMS tracks vary by mass, but:
start with free-fall
central regions start to collapse faster, forms core
accretes more material
finishes accretion and blows off remainder with heat, light pressure