NAME: 

Section: 01(Th 10-10:50) 02(Th 1-1:50) 03(T 12-12:50)

04(T 1-1:50) 05(Th11-11:50)

Mid-Term Exam 1 - PHYS 298

Mendes, Spring 2015, Sept 25

Start time: 10:00 a.m.
End time: 10:50 a.m.

- Calculators allowed; no other electronic device allowed

- You can use during the test the reference card that you prepared (a single sheet of paper, double-sided, 8.5"×11")

- Absolutely no transfer of any sort of material during the test

- In solving the questions, show all the steps of your work and clearly state your final answer

- Where it is appropriate, make sure to provide physical units to your numerical answer

- If needed, consider g = 9.8 m/s²
1) Given two vectors:

\[ \vec{A} = -2.00 \hat{i} + 3.00 \hat{j} + 4.00 \hat{k} \]

and

\[ \vec{B} = 3.00 \hat{i} + 1.00 \hat{j} + 3.00 \hat{k} , \]

do the following:

a) Find the magnitude of each vector.

b) Write an expression for the vector difference \( \vec{A} - \vec{B} \) using unit vectors.

c) Find the scalar product \( \vec{B} \cdot \vec{A} \)

\[ a) \quad | \vec{A} | = \sqrt{A_x^2 + A_y^2 + A_z^2} \]

\[ = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = \sqrt{29.0} \]

\[ | \vec{B} | = \sqrt{B_x^2 + B_y^2 + B_z^2} \]

\[ = \sqrt{(3.00)^2 + (1.00)^2 + (3.00)^2} = \sqrt{19.0} \]

\[ b) \quad \vec{A} - \vec{B} = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j} + (A_z - B_z) \hat{k} = \]

\[ = (-2.00 - 3.00) \hat{i} + (3.00 - 1.00) \hat{j} + \\
+ (4.00 - 3.00) \hat{k} = \]

\[ = -5.00 \hat{i} + 2.00 \hat{j} + 1.00 \hat{k} \]
c) \( \vec{B} \cdot \vec{A} = B_x A_x + B_y A_y + B_z A_z = \)

\( = 3.00 \times (-2.00) + 1.00 \times 3.00 + 3.00 \times 4.00 = \)

\( = 9.00 \)
2) A test car travels in a straight line along the x-axis. The graph below shows the car’s position as a function of time. Answer the following questions:

![Graph showing the car's position as a function of time](image)

a) What is the instantaneous velocity at time $t_1 = 1 \text{ s}$?

b) What is the instantaneous velocity at time $t_2 = 4 \text{ s}$?

c) What is the average velocity for the time interval between $t_1 = 1 \text{ s}$ and $t_2 = 4 \text{ s}$?

d) What is the instantaneous acceleration at time $t_3 = 8 \text{ s}$?

\[ a) \text{ slope at } t_1 = 1 \text{ s} : v_x = \frac{3 \text{ m}}{3 \text{ s}} = 1 \frac{\text{ m}}{\text{ s}} \]

\[ b) \text{ slope at } t_2 = 4 \text{ s} : v_x = \frac{0 \text{ m}}{2 \text{ s}} = 0 \frac{\text{ m}}{\text{ s}} \]

\[ c) \text{ slope between } t_1 \text{ and } t_2 : v_{x, \text{ av}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(3 - 1) \text{ m}}{(4 - 1) \text{ s}} = \frac{2}{3} \frac{\text{ m}}{\text{ s}} = 0.7 \frac{\text{ m}}{\text{ s}} \]
d) at \( t_3 = 8 \text{ s} \), slope = \( v_x \) = constant \( \Rightarrow \) 
\[ a_x = 0 \] (no curvature on the \( x \) versus \( t \) plot)
3) Two crates, one with mass 4.00 kg and the other with mass 6.00 kg sit on the frictionless surface of a frozen pond, connected by a light rope (see Figure below).

A woman wearing golf shoes (so she can get traction on the ice) pulls horizontally on the 6.00-kg crate with a force \( F \) that gives the crate an acceleration of 5.00 m/s\(^2\)

a) What is the acceleration of the 4.00-kg crate?

b) Draw a free-body diagram for the 4.00-kg crate. Use that diagram and Newton’s second law to find the tension \( T \) in the rope that connects the two crates.

c) Draw a free body diagram for the 6.00-kg crate. What is the direction of the net force on the 6.00-kg crate?

d) Use part (c) and Newton’s second law to calculate the magnitude of the force \( F \).

\[ a_{2,x} = 5.00 \text{ m/s}^2 \text{, as the two crates are pulled together.} \]

\[ \sum F_x = T = m_2 a_{2,x} \quad (1) \]

\[ \sum F_y = m_2 g - w_2 = 0 \quad \text{(Equilibrium)} \]

From (1): \[ T = m_2 a_{2,x} = 4.00 \text{ kg} \times 5.00 \text{ m/s}^2 = 20.0 \text{ N} \]
The net force should point to the right (+x-axis) to accelerate to the right crate (1).

c)

\[ \sum F_y = m_1 - w_1 = 0 \]

\[ \sum F_x = F - T = m_1 a_{1,x} \Rightarrow F = T + m_1 a_{1,x} \]

\[ F = 20.0 \text{ N} + 6.00 \text{ Kg} \cdot \frac{5.00 \text{ m}}{s^2} = 50.0 \text{ N} \]

\[ 30.0 \text{ N} \]
4) A tennis ball is thrown with an initial velocity described an upward component of 10 m/s and a horizontal component of 20 m/s. Consider that air resistance is negligible for this tennis ball.

a) How much time is required for the tennis ball to reach the highest point of the trajectory?

b) How high is this point?

c) How much time (after it is thrown) is required for the ball to return to its original level?

d) How far has the ball traveled horizontally during this time interval?

\[ \vec{v}_0 = 20 \frac{m}{s} \hat{i} + 10 \frac{m}{s} \hat{j} \]

\[ \begin{align*}
\text{a) } v_y &= v_{0y} - gt \\
v_y &= 0 \Rightarrow t_{\text{max}} &= \frac{v_{0y}}{g} = \frac{10 \frac{m}{s}}{9.8 \frac{m}{s^2}} = 1.0 \text{ s}
\end{align*} \]

\[ \begin{align*}
\text{b) } y &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\
a_y &= -g, \quad v_{0y} = 10 \frac{m}{s}
\end{align*} \]

\[ y_{\text{max}} - y_0 = \sqrt{h_{\text{max}}} = v_{0y} t_{\text{max}} - \frac{1}{2} g t_{\text{max}}^2 = 10 \frac{m}{s} \times 1.0 \text{ s} - \frac{1}{2} \times 9.8 \frac{m}{s^2} \times (1.0 \text{ s})^2 = 10 \text{ m} - 4.9 \text{ m} = 5.1 \text{ m} \]
c) \( y = y_0 \Rightarrow y_0 = y_0 + v_{0y} t - \frac{1}{2} g t^2 \Rightarrow \)
\[ 0 = t \left( v_{0y} - \frac{1}{2} g t \right) \Rightarrow \]
\[ t = \frac{2v_{0y}}{g} = \frac{2 \times 10 \text{ m/s}}{9.8 \text{ m/s}^2} = 2.0 \text{ s} \]

d) \( x = x_0 + v_{0x} t \)
\[ t = 2.0 \text{ s} \Rightarrow x - x_0 = R = \frac{20 \text{ m}}{g} \times 2.0 \text{ s} = 40 \text{ m} \]