1. What are aberrations?

A lens made of a uniform glass with spherical surfaces cannot form perfect images. Spherical aberration is a prominent image defect for a point source on the optical axis of such a lens. It arises because all rays through the lens are not focused to a common point.

The dependence of index of refraction on wavelength also causes the focal length, and thereby the image position, to depend on the color of the light. All simple lenses suffer from such chromatic aberration. This experiment allows you to see both chromatic aberration, and spherical aberrations for a plano-convex lens, and to measure them approximately.

Spherical aberration

A plano-convex lens, with radius of curvature \( R \), index of refraction, \( n \), and thickness, \( t \), has a focal length, \( f \), that depends on the distance from the optical axis \( h \) that the light rays pass through the lens. The variation of the focal length with distance off the axis is non-linear, and also depends on whether the parallel light enters the lens from the curved or flat side. For example, if parallel light is incident from the plane side, it images on axis at

\[
S = \frac{R}{n-1} - \frac{h^2}{2R} \tag{1}
\]
behind the curved surface. The general form for spherical aberration is

\[
\delta f = \frac{a h^2}{f}
\]

(2)

where \(\delta f\) is the change in the focal length \(f\). You can let \(a = 1\) here. For a plano-convex lens,

\[
f = \frac{R}{n-1}
\]

(3)

**Chromatic aberration**

Equation (3) shows that \(f\) varies as \(n\) varies. Generally, \(n\) is larger for shorter wavelength, \(\lambda\), and it follows that \(f\) is shorter when \(\lambda\) is smaller. Lenses have a shorter focal length for blue light than for red. The change in \(f\) can be found by differentiating Equation (3)

\[
\frac{\delta f}{f} = \frac{-\delta n}{n-1}
\]

(4)

**Evaluating aberrations with a Lens Design Software**

It is often easiest to follow image defects in an optical system with the help a ray tracing program. The one we have is called OSLO and you can download it for free at the following website http://www.lambdares.com/buy/educators-and-students, and install it in your computer. It allows you to select several incoming light rays, adjust the curvature and placement of lens surfaces, and observe where the rays actually will go after refraction. Ray tracing programs are
as good as an experiment; they actually calculate the effect of refraction at each surface on each individual light ray, rather than rely on idealized concepts like focal length and paraxial rays.

2. Observations of spherical and chromatic aberrations

Place carriers on the bench

There are only four carriers on the optical bench for this experiment. The zirconium point arc lamp is at the far end, near zero on the scale. A finely ground focusing screen is at the other end, near the 100 cm mark. The lens you will test is in a carrier on the bench. Do not remove the lens from its holder. At most, you will just have to rotate it 180° to place either the curved or flat side toward the arc. Check, however, to be sure that the lens is at the same height as the point arc; if not, make the necessary adjustment to the height of the lens holder. Leave the point arc alone. A fourth carrier is provided to hold the lens mask. The arrangement of components is illustrated by Figure 1.

Measure the focal length

Put the curved side of the lens toward the arc, and move it slowly away from the focusing screen. You should notice that when it is about 8 cm from the screen the image will be as small as possible. Locate the lens at the position of best focus. Make note of the distance from the index mark of each carrier (arc, lens, and screen) to each respective element, and find the distances from the arc to the lens (s) and from the lens to the screen (s'). You will use this simple measurement to evaluate the focal length of the lens.
Measure the spherical aberration

There are aluminum masks with holes 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 cm apart. These masks provide beams passing 0.25, 0.5, ... cm off the axis. Select the 2.5 cm mask and place it in a carrier close to the lens, between the lens and the arc. Be careful not to move the lens. Also take care that the carrier for these masks is at the correct height. Look closely at the focusing screen (you may need to use the magnifier) and you will notice two small dots of light. These are the places where the beams passing through the two holes hit the screen. If you move the lens slightly along the bench, you can superimpose the dots. Since these spots are less than 1 mm across, it does help to use a magnifier to get the best focus. Carefully read the position of the carrier for the lens by making use of the vernier accuracy of 0.1 mm. Before continuing, make sure that all the carriers except the one for the screen are now firmly locked in place. Carefully remove the 2.5 cm mask and insert the 0.5 cm mask. Slide the screen to its new position of best focus and note the new reading with the vernier. The change in the screen position is $\delta f$. Measure and tabulate the positions for each mask. With the value for the 0.5 cm mask as reference, find $\delta f$ for the others. The values of $\delta f$ should be associated with $h$, that is with 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, and 2.0 cm.

Orient the lens with flat side toward lamp

Rotate the holder for the test lens 180° to place the curved lens surface away from the arc and toward the focusing screen. Go through the sequence of mask measurements again (there is no need to repeat the measurement of $f$).
Colors

By now you will have noticed that each image dot is actually a short spectrum. Carefully inspect the images with the 2.5 cm mask and with the lens oriented so that its curved side is toward the arc. Slide it toward the focusing screen until the ideal focus is on the eye side of the screen with the lens too close to the screen for a good focus. Is the red end of the spectrum closer to the axis than the blue end, or farther away?

Now very slowly adjust the lens so that the red parts of each dot just touch. This is the best focus for red light. Read the lens position precisely. Move the lens and bring the blue parts of each image into contact. Read the lens position again. The difference in these two positions is the longitudinal chromatic aberration of the lens. It will very nearly be the difference in the focal lengths of the lens for red and blue light since the arc source is much farther from the lens than the focusing screen.
Figure 1: The zirconium arc point source at the left illuminates a mask containing two holes in front of a lens on the right. The lens images the point source on the screen. If the screen is not exactly at the focus, the two beams through the lens leave two dots on the screen. The location of the focal plane for the part of the lens not blocked is found by precisely moving the screen to get just one dot.
3. Analyze the observations

Find the focal length

Calculate the focal length of the lens from the measurement of $s$ and $s'$ by using the lens equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$  \hspace{1cm} (5)

This lens is made from fused silica. Its average index of refraction for visible light is 1.46. Calculate the radius of curvature of the lens from Equation (3).

Observe the characteristic $h^2$ dependence

You should enter your data to a computer file and use your preferred software (e.g. Excel) to plot your results. Plot a graph of $\delta f \times f$ versus $h$, and versus $h^2$ for the two orientations of the lens. Also on this graph plot the theoretical curve evaluated from Equation (2).

The best way to use a plano-convex lens

For which orientation of the plano-convex lens is spherical aberration minimized when imaging nearly parallel light? Since many simple telescopes and binoculars use plano-convex achromats as their objective lenses (the big lens closest to the object), these lenses must always be oriented in this way to produce good images.
Color in images

The index of refraction is slightly larger for blue light than for red. How does this explain the order of the colors seen in the chromatic aberration experiment?

Ray tracing

Use the OSLO ray tracing program to compute the spherical and chromatic aberrations for this lens. The information on the lens (radius of curvature, material = fused silica, center thickness) and on the rays will need to be entered in the program to describe your experiment. Ask the software to trace several rays under conditions similar to your experiment, and plot diagrams to evaluate the imaging capability of your lens. Make a paper copy of the ray trace and spot diagrams. These should be included with your lab report.