2) A 1 gram block oscillates back and forth along a straight line on a frictionless horizontal surface. It's displacement from the origin is given by,

$$x = 5\ (\text{mm})\cos[100\pi(\text{rad/s})t + \pi/4(\text{rad})]$$

(a) What is the oscillation frequency ? (4)

$\omega = 100\pi$ rad/s $\quad \omega = 2\pi\nu$

$\Rightarrow \nu = \omega/2\pi = 100\pi/2\pi = 50$ Hz (s^{-1})

(b) What is the maximum speed acquired by the block ? At what value of x does this occur ? (7)

$x = 5\times10^{-3}\cos(100\pi t + \pi/4)$

$V_x = \frac{dx}{dt} = -100\pi\, 5\times10^{-3}\sin(100\pi t + \pi/4)$

$\Rightarrow |V_x^{MAX}| = 5\pi\, 10^{-1} = 0.5\pi$ m/s

occurs when $\sin(100\pi t + \pi/4) = \pm 1$ or $100\pi t + \pi/4 = \pm\pi/2$

$\Rightarrow x = 5\times10^{-3}\cos(\pm\pi/2)$

$x = 0$

(c) What is the maximum acceleration of the block ? At what value of x does this occur ? (7)

$a_x = dv_x/dt = -10000\pi^2\, 5\times10^{-3}\cos(100\pi t + \pi/4)$

a_x^{MAX} occurs when cosine in above expression is equal to -1

then $a_x^{MAX} = 10^4\pi^2\, 5\times10^{-3} = 50\pi^2$ m/s^2

occurs when $\cos(100\pi t + \pi/4) = -1$

$\Rightarrow x = -5\times10^{-3}$ m $\quad (-5$ mm$)$

(d) What force, applied to the block, results in the given oscillation ? (7)

$F = -kx$ where $k = m\omega^2$

$= 10^{-3}\times10000\pi^2 = 10\pi^2$ N/m

$\Rightarrow F = -10\pi^2 x$ N

where x is in metres