Summary of critical formulas and important facts

Physics 298 – Test 2

Work and Power

• Constant Force:

 $W=\vec{F}\bullet\vec{d}$

• Variable 1D force:

$$W = \int_{x_1}^{x_2} F(x) dx$$
 = Area under F(x) between x₁ and x₂

• Power:

Instantaneous Power = dW/dt

Average Power = W_{tot}/t

 $P = \vec{F} \bullet \vec{v}$ (constant **F**)

Hooke's Law : Stretched/Compresed Spring

• Restoring Force:

 $F_R = -kx$

• Work:

$$W = \frac{1}{2}kx^2$$

Energy

- Kinetic Energy:
- $KE = \frac{1}{2}mv^2$
- Potential Energy:
 - o Gravitational

 $U_g = mgh$

o Elastic

 $U_e = \frac{1}{2}kx^2$

Work Energy and Conservation of Energy

• Work energy theorem:

$$W = \Delta K = K_f - K_i$$

• Conservation of energy:

$$\Delta U + \Delta K = W_{NC}$$

- For conservative forces only, $\Delta U + \Delta K = 0$, or U + K, the mechanical energy is constant
- The position of $U_g = 0$ is arbitrary

Centre of Mass

$$\vec{r}_{cm} = \frac{\sum_{i} m_{i} \vec{r}_{i}}{\sum_{i} m_{i}}$$

Momentum and Collisions

• Momentum

$$\vec{p} = m\vec{v}$$

• 2nd Law, alternate form

$$\vec{F} = \frac{d\vec{p}}{dt}$$

• Impulse

$$\vec{I} = \vec{F} \ \Delta t = \Delta \vec{p}$$

• With no external force on a system, momentum is conserved (independently in xyz)

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = 0$$

• Elastic collisions : KE conserved

• Inelastic collision: KE not conserved. In a completely inelastic collision the colliding object stick together.

Rotational kinematic equations

 $\omega_{f} = \omega_{i} + \alpha t$ $\theta = \frac{1}{2}(\omega_{i} + \omega_{f})t$ $\theta = \omega_{i} t + \frac{1}{2}\alpha t^{2}$ $\omega_{f}^{2} = \omega_{i}^{2} + 2\alpha\theta$

<u>Rotational – Translational a and v</u>

$$v = \omega R$$
$$a_T = \alpha R$$

• Centripetal acceleration

$$a_C = a_{Rad} = \frac{v^2}{R} = \omega^2 R$$

Rotational Dynamics

• Torque:

$$\underline{\tau} = \underline{r} \times \underline{F}$$
$$|\underline{\tau}| = |\underline{r}||\underline{F}|\sin\theta_{rF} = |\underline{r}|F_{\perp} = |\underline{F}|r_{\perp}$$

• Angular momentum:

$$\underline{\ell} = \underline{r} \times \underline{p}$$
$$|\underline{\ell}| = |\underline{r}||\underline{p}|\sin\theta_{p}| = |\underline{r}|p_{\perp}| = |\underline{p}|r_{\perp}|$$

• Moment of inertia:

$$I = \sum_{i} m_{i} r_{i}^{2}$$

• Parallel axis theorem:

$$I_P = I_C + Mh^2$$

• Kinetic energy:

$$K = \frac{1}{2}I\omega^2$$

• Angular momentum:

 $L = I\omega$

• Power:

Power =
$$\tau \omega$$

• Work:

$$W = \int \tau \, d\theta = \tau \, \Delta \theta$$

• 2^{nd} Law:

$$\tau = I\alpha = \frac{dL}{dt}$$

• Angular momentum conservation:

$$L_i = L_f$$
$$I_i \omega_i = I_f \omega_f$$

Remember, $I_i \neq I_f$ in many cases