25 Reflection and Refraction of Plane Waves

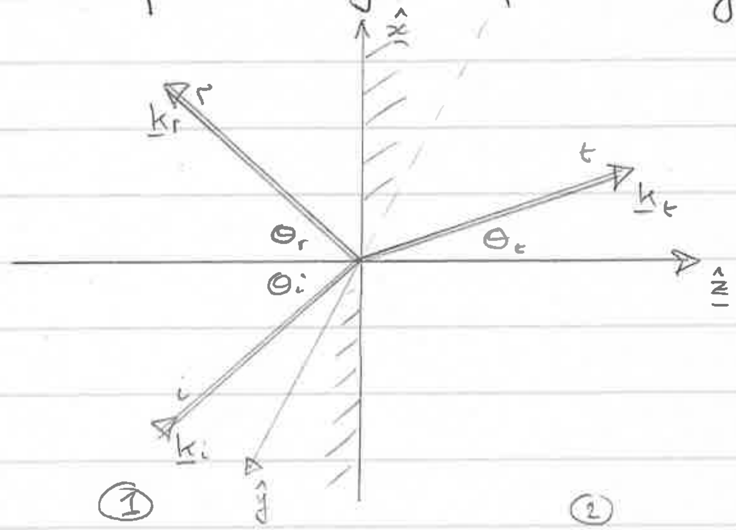
* This is a long chapter and as such it makes sense to list the topics it covers before beginning so that you have some idea of the reasons why we are looking at various quantities.

1) Laws of reflection and refraction [Familiar in OPTICS]
   (a) Incident, reflected and transmitted rays are in the same plane
   (b) Incident, reflected and transmitted rays have same frequency.
   (c) $\Theta_i = \Theta_r$
   (d) Snells Law $n_1 \sin\Theta_i = n_2 \sin\Theta_t$
   (e) Total internal reflection and critical angle.
2) Ratios of (E_r/E_i) and (E_t/E_i) in two different scenarios (a) $E_\perp$ to plane of incidence and (b) E in the plane of incidence
   ⇒ reflection and transmission coefficients or what percentage of energy is reflected and transmitted.
3) Phase changes on reflection and Brewsters Law (polarisation of light on reflection)
4) What happens in the "other" medium when TIR takes place?
5) Reflection from conductors [Maybe leave this till conductors are discussed]
6) Radiation pressure.

* In all cases, unless specified otherwise we will assume

(i) Boundary is a plane infinite surface
(ii) No free charges or currents on the surface ($\rho_f = J_f = 0$)
(iii) Both media are l.i.h. in magnetic and electric properties (as we assumed in chapter 24)
(iv) $\sigma = 0$ — non-conducting media
[N.B. the conducting case may be considered later]
(v) We assume <u>plane</u> waves incident, leading to a reflected and transmitted <u>plane</u> wave after interaction with the boundary.

* Imagine a plane wave incident at angle Θ_i on the boundary between two surfaces (xy plane forms boundary)



Θ_i Θ_r Θ_t are as defined.

The $\underline{E}$ vector of the waves may be written

$$\underline{E}_i = \underline{E}_{0i}\, e^{i(\underline{k}_i \cdot \underline{r} - \omega_i t)}$$

$$\underline{E}_r = \underline{E}_{0r}\, e^{i(\underline{k}_r \cdot \underline{r} - \omega_r t)}$$

$$\underline{E}_t = \underline{E}_{0t}\, e^{i(\underline{k}_t \cdot \underline{r} - \omega_t t)}$$

The boundary conditions from Maxwells equations (for $\underline{E}$) lead to

$$\left[\underline{E}_i + \underline{E}_r\right]_{\text{tangential}} = \left[\underline{E}_t\right]_{\text{tangential}}.$$ (on surface)

for all values of t and any value of $\underline{r}$ [$\underline{r}$ is the position vector of the point of incidence w.r.t some arbitrary origin] (Origin is defined at pt on xy plane)

t appears only in the exponential and since the b.c is true for all t we must have

$$A e^{-i\omega_i t} + B e^{-i\omega_r t} = C e^{-i\omega_t t}$$

(A, B, C independent of t.)

The only way to ensure this equality for all t given fixed values of $\underline{r}$ is if $\omega_i = \omega_r = \omega_t$

$\Rightarrow$ $\boxed{\text{frequencies of incident, reflected and transmitted rays are equal}}$

* With this equality we may now write the bc as

$$\left[\underline{E}_{oi} e^{i(\underline{k}_i \cdot \underline{r})} + \underline{E}_{or} e^{i(\underline{k}_r \cdot \underline{r})}\right]_{tang} = \left[\underline{E}_{ot} e^{i(\underline{k}_t \cdot \underline{r})}\right]_{tang}$$

Once again, the only way we can satisfy this for all $\underline{r}$ on the xy plane is if

$$\underline{k}_i \cdot \underline{r} = \underline{k}_r \cdot \underline{r} = \underline{k}_t \cdot \underline{r}$$

On the xy plane $\underline{r} = \hat{\underline{x}} x + \hat{\underline{y}} y \qquad (z = 0)$

$\Rightarrow$

$$\left.\begin{aligned} \underline{k}_i \cdot \underline{r} &= k_{ix} x + k_{iy} y \\ \underline{k}_r \cdot \underline{r} &= k_{rx} x + k_{ry} y \\ \underline{k}_t \cdot \underline{r} &= k_{tx} x + k_{ty} y \end{aligned}\right\} \text{must all be equal for all } x \text{ and all } y.$$

$\Rightarrow$ at $y = 0$

$$k_{ix} = k_{rx} = k_{tx} \quad \{\text{must also be true for all } y\}$$

and at $x = 0$

$$k_{iy} = k_{ry} = k_{ty} \quad \{\text{must also be true for all } x\}$$

But if we define $k_{iy} = 0$ – incident wave in the xz plane (as in the diagram) – then $k_{ry} = k_{ty} = 0$

$\Rightarrow$ $\underline{k}_r$ and $\underline{k}_t$ can have only x and z components – in other words

$\boxed{\text{Incident, reflected and transmitted waves are in the same plane}}$

* Now $|\underline{k}_i| = \omega / v_i = \dfrac{\omega n_1}{c}$ $(n_1 = c/v_1)$

$$|\underline{k}_r| = \omega / v_r = \frac{\omega n_1}{c} = |\underline{k}_i|$$

$$|\underline{k}_t| = \omega / v_t = \frac{\omega n_2}{c}$$

$$\Rightarrow \quad \underline{k}_i = \frac{\omega n_1}{c}\left(\hat{\underline{x}} \sin\Theta_i + \hat{\underline{z}} \cos\Theta_i\right)$$

$$\underline{k}_r = \frac{\omega n_1}{c}\left(\hat{\underline{x}} \sin\Theta_r - \hat{\underline{z}} \cos\Theta_r\right)$$

$$\underline{k}_t = \frac{\omega n_2}{c}\left(\hat{\underline{x}} \sin\Theta_t + \hat{\underline{z}} \cos\Theta_t\right)$$

But we have just shown that $k_{ix} = k_{rx} = k_{tx}$ for any y.

$$\Rightarrow \quad \underbrace{n_1 \sin\Theta_i = n_1 \sin\Theta_r}_{\Theta_i = \Theta_r} = n_2 \sin\Theta_t$$

$$\boxed{\text{Angle of incidence} = \text{angle of reflection}}$$

* and $\boxed{n_1 \sin\Theta_i = n_2 \sin\Theta_t \quad \text{— Snell's Law}}$

* Note that in deriving the four "boxed" relationships all we have used of Maxwells equations is the fact that the components of $\underline{E}$ tangential to the boundary surface are continuous. This type of condition is satisfied <u>also</u> for water, sound etc waves. Electrodynamics only enters the problem in a significant way when we consider the relative amplitudes of the incident, reflected, and transmitted

waves. However, before considering these amplitudes there is one remaining issue to be addressed.

* Total Internal Reflection:

$$\sin\theta_t = (n_1/n_2)\sin\theta_i$$

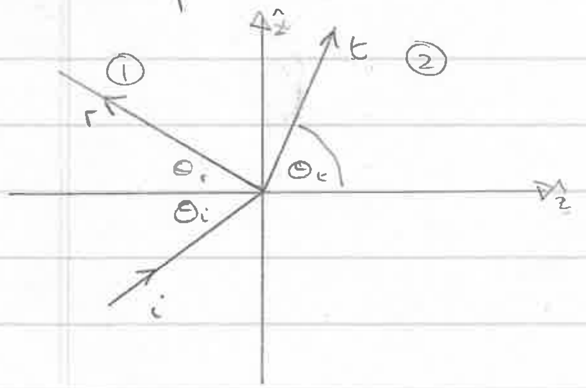
For $n_2 > n_1$ whatever the value of θ_i ($\sin\theta_i$) $\sin\theta_t$ will always satisfy $0 \leq \sin\theta_t \leq 1$ [i.e. there will always be a transmitted wave]

But what if $n_1 > n_2$, then $(n_1/n_2) > 1$ and there will be some values of θ_i for which $\sin\theta_t > 1$

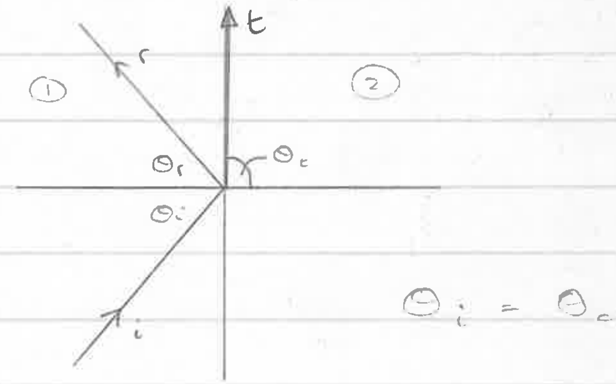
The value of θ_i for which $\sin\theta_t = 1$ is called the critical angle θ_c

$$\sin\theta_c = n_2/n_1$$

for $\theta_i < \theta_c$ then $\theta_t > \theta_i$ and $\sin\theta_t < 1$
and for $\theta_i = \theta_c$ " $\theta_t = \pi/2$ " $\sin\theta_t = 1$



$n_1 > n_2$



$n_1 > n_2$

For cases where $n_1 > n_2$ and $\theta_i > \theta_c$

$$\underline{E}_t = \underline{E}_{0t}\, e^{i(\underline{k}_t\cdot\underline{r} - \omega t)}$$
$$= \underline{E}_{0t}\, e^{i(k_{tx}x + k_{ty}y + k_{tz}z - \omega t)}$$

But incident plane has $y = 0$ and $k_{tz} = \dfrac{\omega n_2 \cos\theta_t}{c}$

and $\quad k_{tx} = \frac{\omega n_2 \sin\theta_t}{c} = \frac{\omega n_1 \sin\theta_i}{c}$

$$\Rightarrow \quad \underline{E}_t = \underline{E}_{0t}\, e^{ix\frac{\omega n_1}{c}\sin\theta_i - i\omega t}\, e^{i\frac{\omega n_2}{c}\cos\theta_t z}$$

What about $\cos\theta_t$?

$$\cos^2\theta_t = 1 - \sin^2\theta_t = 1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i$$

$$= 1 - \underbrace{\left(\frac{\sin^2\theta_i}{\sin^2\theta_c}\right)}_{>1 \quad \sin\theta_c < \theta_i} < 0$$

$$\cos^2\theta_t = -\left[\left(\frac{\sin^2\theta_i}{\sin^2\theta_c}\right) - 1\right]$$

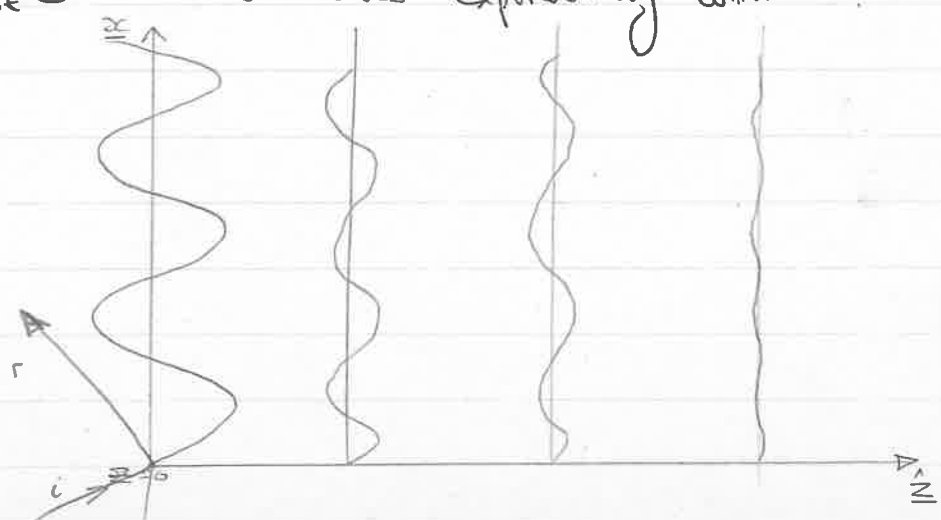
$$\cos\theta_t = \pm i \left[\frac{\sin^2\theta_i}{\sin^2\theta_c} - 1\right]^{1/2}$$

$$\Rightarrow \quad \underline{E}_t = \underline{E}_{0t}\, e^{ix\frac{\omega n_1}{c}\sin\theta_i - \omega t i}\, \underbrace{e^{\mp \frac{\omega n_2}{c}\left[\frac{\sin^2\theta_i}{\sin^2\theta_c} - 1\right]^{1/2} z}}_{\text{negative sign provides only physical solution.}}$$

With $\quad k = \frac{n_2\omega}{c}\left[\left(\frac{\sin\theta_i}{\sin\theta_c}\right)^2 - 1\right]^{1/2}$

$$\underline{E}_t = \underline{E}_{0t}\, e^{-kz}\, e^{i(k_1 x \sin\theta_i - \omega t)}$$

which describes a wave moving in the x-direction whose amplitude - $\underline{E}_{0t} e^{-kz}$ - decreases exponentially with z.



$\hat{\underline{z}}$ scale greatly exaggerated

The typical depth (in z) of the wave is given by $\delta = 1/\kappa$

$$\delta = \frac{c}{n_2 \omega}\left(\frac{\sin^2\Theta_i \, n_1^2}{n_2^2} - 1\right)^{-1/2} = \frac{1}{k_2}\frac{1}{\left[\left(n_1/n_2\right)^2 \sin^2\Theta_i - 1\right]^{1/2}}$$

$$v = \frac{\omega}{k} = \frac{c}{n} = \frac{\omega\lambda}{2\pi} \qquad \frac{c}{n\omega} = \frac{\lambda}{2\pi}$$

$$= \frac{(\lambda_2/2\pi)}{\left[(\sin^2\Theta_i/\sin^2\Theta_c) - 1\right]^{1/2}}$$

but typically less than unity.

remember for TIR $\Theta_i > \Theta_c \Rightarrow$ denominator above is > 0 and δ will be typically a few wavelengths (for Θ_i significantly $> \Theta_c$). Note that when $\Theta_i \sim \Theta_c$ the penetration depth δ rapidly increases – as it must, for when $\Theta_i < \Theta_c$ there is a "real" transmitted wave.

We can also show that the speed of this wave in x-direction

$$k_{tx} = \frac{\omega n_2 \sin\Theta_c}{c} = \frac{\omega n_1 \sin\Theta_i}{c}$$

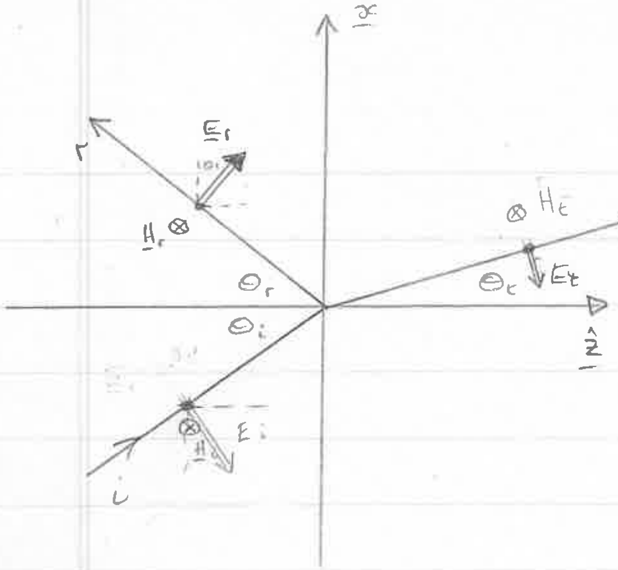
$$v_{2x} = \omega/k_{tx} = \frac{c}{n_1 \sin\Theta_i} = \frac{c/n_2}{n_1/n_2 \sin\Theta_i} = \underbrace{\left(\frac{\sin\Theta_c}{\sin\Theta_i}\right)}_{<1 \ (\Theta_c < \Theta_i)} v_2$$

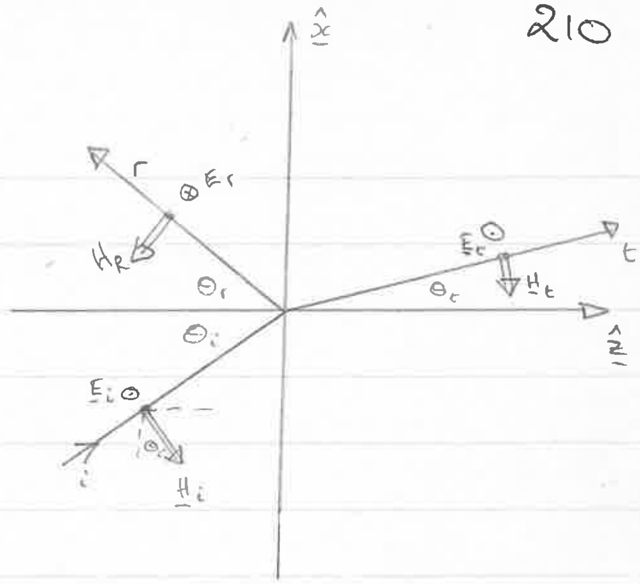
thus the transmitted wave – <u>the evanescent wave</u> – travels more slowly than the usual plane wave velocity v_2.
(in medium (2))

* <u>Relative amplitudes (and intensities) of Incident, Reflected and Transmitted waves</u>:

The behaviour of the $\underline{E}$ field is different depending on whether $\underline{E}$ is in the plane of incidence or perpendicular to the plane of incidence. [All other polarisations can be reduced to a linear combination of these two cases]

The two situations we will consider are shown in the diagrams below





$\underline{E}_i$ in plane of incidence | $\underline{E}_i$ @ 90° to plane of incidence.

[N.B. we can obtain the direction of $\underline{E}, \underline{H}$ by using $\underline{S} = \underline{E} \wedge \underline{H}$ – $\underline{S}$ is in the direction of energy flow]

$$\underline{E}_i = \underline{E}_{oi}\, e^{i(\underline{k}_i \cdot \underline{r} - \omega t)}$$ ⇒ Drop the ωt dependence for simplicity

$$= \underline{E}_{oi}\, e^{\frac{i n_1 \omega}{c}(x \sin\theta_i + z\cos\theta_i)}$$

Therefore

$$E_{xi} = -E_{oi}\cos\theta_i \exp[\quad]$$
$$E_{zi} = E_{oi}\sin\theta_i \exp[\quad]$$

Similarly

$$H_{yi} = -H_{oi} \exp[\quad]$$

$$\underline{E}_i = \underline{E}_{oi}\, e^{\frac{i n_1 \omega}{c}(x \sin\theta_i + z\cos\theta_i)}$$

so that

$$E_{yi} = E_{oi} \exp[\quad]$$
$$H_{xi} = -H_{oi}\cos\theta_i \exp[\quad]$$
$$H_{zi} = H_{oi}\sin\theta_i \exp[\quad]$$

But $|\underline{E}|/|\underline{H}| = Z$ where $Z = (\mu/\varepsilon)^{1/2}$

Therefore with $E_{oi} = E_i$ and $H_{oi} = H_i$

$$E_{xi} = -E_i \cos\theta_i \exp[\quad] \qquad E_{yi} = E_i \exp[\quad]$$
$$E_{zi} = E_i \sin\theta_i \exp[\quad] \qquad H_{xi} = -(E_i/Z_1)\cos\theta_i \exp[\quad]$$
$$H_{yi} = -(E_i/Z_1) \exp[\quad] \qquad H_{zi} = (E_i/Z_1)\sin\theta_i \exp[\quad]$$

For the reflected wave we obtain

$$E_{xr} = E_r \cos\theta_r\, e^{\frac{i n_1 \omega}{c}(x\sin\theta - z\cos\theta)} \qquad E_{yr} = -E_r \exp\left[\frac{i n_1 \omega}{c}(x\sin\theta - z\cos\theta)\right]$$
$$E_{zr} = E_r \sin\theta_r \exp[\quad] \qquad H_{xr} = -(E_r/Z_1)\cos\theta_r \exp[\quad]$$
$$H_{yr} = -(E_r/Z_1) \exp[\quad] \qquad H_{zr} = -(E_r/Z_1)\sin\theta_r \exp[\quad]$$

and finally for the transmitted wave

$$E_{xt} = -E_t \cos\Theta_t \, e^{\underbrace{\frac{i n_2 \omega}{c}(x\sin\Theta_t + z\cos\Theta_t)}}$$

$$E_{zt} = E_t \sin\Theta_t \exp[\quad]$$

$$H_{yt} = -(E_t/Z_2)\exp[\quad]$$

$$E_{yt} = E_t \exp\left[\underbrace{\frac{i n_2 \omega}{c}(x\sin\Theta_t + z\cos\Theta_t)}\right]$$

$$H_{xt} = -(E_t/Z_2)\cos\Theta_t \exp[\quad]$$

$$H_{zt} = (E_t/Z_2)\sin\Theta_t \exp[\quad]$$

Maxwells equations tell us that the tangential components of $\underline{E}$ and $\underline{H}$ are continuous across a boundary.

Therefore at $z = 0$ (all exponentials are equal at $z=0$)

x component:

$$-E_i \cos\Theta_i + E_r \cos\Theta_r = -E_t \cos\Theta_t$$

y component:

$$-(E_i/Z_1) - (E_r/Z_1) = -(E_t/Z_2)$$

y component

$$E_i - E_r = E_t$$

x component

$$-(E_i/Z_1)\cos\Theta_i - (E_r/Z_1)\cos\Theta_r = -(E_t/Z_2)\cos\Theta_t$$

Now in both cases $\Theta_i = \Theta_r = \Theta$ and we obtain

$$\cos\Theta - (E_r/E_i)\cos\Theta = (E_t/E_i)\cos\Theta_t$$

$$Z_2 + (E_r/E_i)Z_2 = (E_t/E_i)Z_1$$

$$1 - (E_r/E_i) = (E_t/E_i)$$

$$-Z_2\cos\Theta - (E_r/E_i)Z_2\cos\Theta = -(E_t/E_i)Z_1\cos\Theta_t$$

Solving for (E_r/E_i) and (E_t/E_i)

this is different from text due to initial assumptions or direction of E_i, E_r ...

$$\underline{\left(\frac{E_r}{E_i}\right) = \frac{-(Z_2\cos\Theta_t - Z_1\cos\Theta)}{(Z_2\cos\Theta_t + Z_1\cos\Theta)}}$$

$$\underline{\left(\frac{E_r}{E_i}\right) = \frac{-(Z_2\cos\Theta - Z_1\cos\Theta_t)}{(Z_2\cos\Theta + Z_1\cos\Theta_t)}}$$

$$\underline{\left(\frac{E_t}{E_i}\right) = \frac{2Z_2\cos\Theta}{Z_2\cos\Theta_t + Z_1\cos\Theta}}$$

$$\underline{\left(\frac{E_t}{E_i}\right) = \frac{2Z_2\cos\Theta}{Z_2\cos\Theta + Z_1\cos\Theta_t}}$$

These equations can be written in other forms using $Z = (\mu/\varepsilon)^{1/2}$ and $n = c/v$ and $v = 1/\sqrt{\mu\varepsilon}$.

We can also obtain expressions for normal incidence and for non-magnetic media, where $\mu_1 = \mu_2$.

I will consider the one special case of $\mu_1 = \mu_2$

then

$\underline{E}$ in plane of incidence

$$\left(\frac{E_r}{E_i}\right) = \frac{\tan(\Theta_i - \Theta_t)}{\tan(\Theta_i + \Theta_t)}$$

$$\left(\frac{E_t}{E_i}\right) = \frac{2\cos\Theta_i \sin\Theta_t}{\sin(\Theta_i + \Theta_t)\cos(\Theta_i - \Theta_t)}$$

$\underline{E}$ perpendicular to plane of incidence

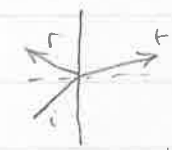
different from text, see previous comments.

$$\left(\frac{E_r}{E_i}\right) = + \frac{\sin(\Theta_i - \Theta_t)}{\sin(\Theta_i + \Theta_t)}$$

$$\left(\frac{E_t}{E_i}\right) = \frac{(n_1/n_2)\sin 2\Theta_i}{\sin(\Theta_i + \Theta_t)}$$

FRESNEL EQUATIONS

* For $n_1 < n_2$ (e.g. light incident from air on glass)



$\Theta_i > \Theta_t$

(E_r/E_i) passes through zero, when $\Theta_i + \Theta_t = \pi/2$, in this case

$n_1 \sin\Theta_i = n_2 \sin\Theta_t$

$n_1 \sin\Theta_i = n_2 \sin(\pi/2 - \Theta_i)$

$\Rightarrow \tan\Theta_i = n_2/n_1$

this is known as Brewsters angle at which time $E_r = 0$

For $\Theta_i < \Theta_B$ $(E_r/E_i) > 0$ and there is no phase change

For $\Theta_i > \Theta_B$ $(E_r/E_i) < 0$ phase change on reflection ($\Theta_t < \Theta_i$)

$(E_t/E_i) > 0$ always

transmitted wave undergoes no phase change

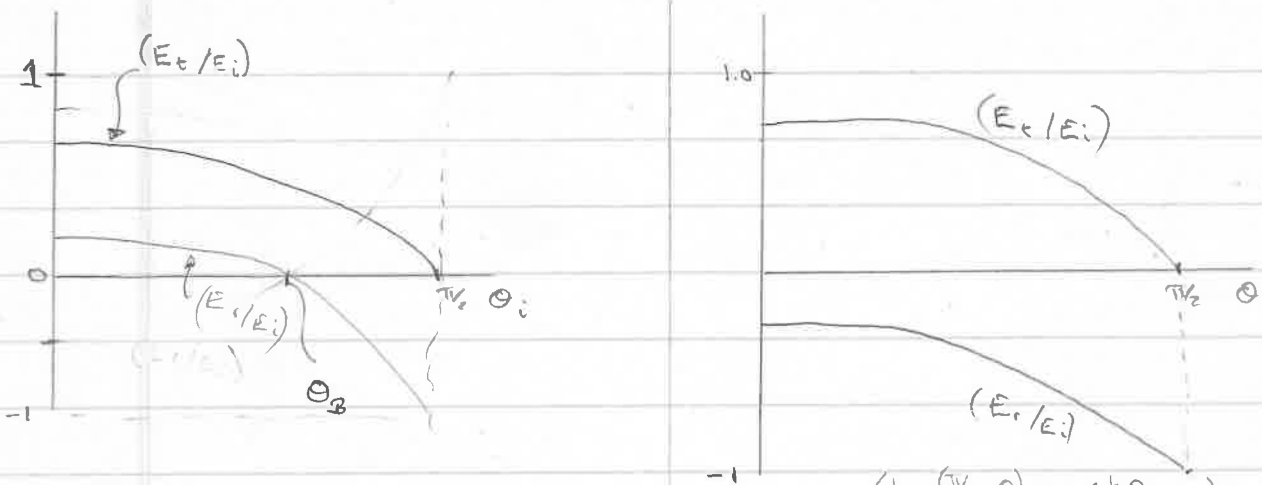
$(E_r/E_i) > 0$

Due to initial direction assumption

$\Rightarrow$ $\underline{E}_r$ is always opposite to $\underline{E}_i$ – in other words $\underline{E}_r$ is changed in phase by 180° w.r.t to $\underline{E}_i$. (phase change on reflection)

$(E_t/E_i) > 0$

Transmitted wave undergoes no phase change



Remember when $\theta_i \sim \pi/2$ θ_t is $< \pi/2$ but finite.

$\left(\frac{\tan(\pi/2 - \theta)}{\tan(\pi/2 + \theta)} = \frac{\cot\theta}{-\cot\theta} = -1 \right)$

Conclusion on phase change:

(a) Transmitted wave never undergoes phase change

(b) Reflected wave undergoes phase change <u>except</u> for $\theta_i < \theta_B$ and $\underline{E}$ vector is in plane of incidence.

Brewsters Law:

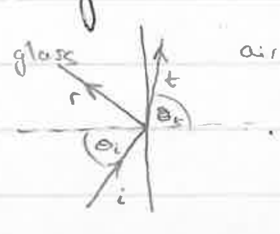
When $\theta_i = \theta_B$ there is no reflected ray with $\underline{E}$ in plane of incidence $\Rightarrow$ for incident unpolarised light at Brewsters angle reflected wave is polarised linearly perpendicular to the plane of incidence.

Grazing Angles:

Notice that in both cases above when θ_i is $90°$ $(E_t/E_i) = 0$. That is <u>all</u> the wave is reflected e.g. glare from headlights of oncoming cars on wet road.

* For $n_1 > n_2$ (e.g. light incident from glass into air)

$\theta_i < \theta_t$



We have already seen in this situation that when $\theta_i > \theta_c$, where θ_c is the critical angle — $\sin\theta_c = n_2/n_1$ ($\sim 41°$ for glass), the wave undergoes TIR. The disturbance in the second medium decreases exponentially with the penetration depth. Let us now investigate the behaviour of the ratios (E_r/E_i) and (E_t/E_i)

E in plane of incidence

$$\left(E_r/E_i\right) = -\left(\frac{Z_2\cos\theta_t - Z_1\cos\theta_i}{Z_2\cos\theta_t + Z_1\cos\theta_i}\right)$$

$$\left(E_t/E_i\right) = \frac{2Z_2\cos\theta_i}{\left(Z_2\cos\theta_t + Z_1\cos\theta_i\right)}$$

E perpendicular to plane of incidence

$$\left(E_r/E_i\right) = -\left(\frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}\right)$$

$$\left(E_t/E_i\right) = \frac{2Z_2\cos\theta_i}{\left(Z_2\cos\theta_i + Z_1\cos\theta_t\right)}$$

* When $\theta_i < \theta_c$ θ_t is real and $\theta_t > \theta_i$ $\theta_t \leq \pi/2$

and for $\mu_1 = \mu_2$

For glass θ_B $(\tan^{-1} n_2/n_1) = 52°$ and θ_c $(\sin^{-1} n_2/n_1) = 41°$

$$\left(E_r/E_i\right) = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

For $\theta_i < \theta_c$ $\theta_i < \theta_B$

When $\theta_i < \theta_B$ $\tan(\theta_i + \theta_t) > 0$

$\Rightarrow \frac{E_r}{E_i} < 1$ $\Rightarrow$ phase change

$$\left(E_r/E_i\right) = +\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

which when included with initial assumption of E_r and E_i opposite gives

$\Rightarrow \left(E_r/E_i\right) < 0$

no phase change on reflection always

(ever, since $\sin(\theta_i + \theta_t) > 0$ always)

Thus for cases where $\theta_i < \theta_c$ and $n_1 > n_2$ there is a phase change on reflection for E in plane of incidence but no phase change for E @ 90° to plane of incidence

[Similarly we can show that the transmitted wave also undergoes no phase change]

* When $\theta_i > \theta_c$ θ_t imaginary, we have

p 208 in notes

$$\cos\theta_t = +ick/n_2\omega \qquad \text{where } k = \frac{n_2\omega}{c}\left[\left(\frac{\sin^2\theta_i}{\sin^2\theta_c}\right)-1\right]^{1/2}$$

$= 1/\delta$ (δ penetration depth)

$$\left(E_r/E_i\right)_\parallel = \frac{iZ_2\left(ck/n_2\omega\right) - Z_1\cos\theta_i}{iZ_2\left(ck/n_2\omega\right) + Z_1\cos\theta_i} = +\frac{(C-iD)}{(C+iD)} = +\frac{(C^2+D^2)^{1/2}e^{-i\phi_\parallel}}{(C^2+D^2)^{1/2}e^{i\phi_\parallel}} = +e^{-2i\phi_\parallel} = e^{-i2\phi_\parallel}$$

$\tan\phi_\parallel = D/C$

$$\left(E_r/E_i\right)_\perp = -\left(\frac{Z_2\cos\theta_i - iZ_1 ck/n_2\omega}{Z_2\cos\theta_i + iZ_1 ck/n_2\omega}\right) = -\left(\frac{A-iB}{A+iB}\right) = \frac{(A^2+B^2)^{1/2}e^{-i\phi_\perp}}{(A^2+B^2)^{1/2}e^{i\phi_\perp}} = e^{-2i\phi_\perp}$$

$\tan\phi_\perp = B/A$

In other words $|E_r/E_i| = 1 \Rightarrow$ reflected amplitude has same magnitude as incident wave. The wave undergoes TIR. The phase factors above imply that the reflected wave is out of phase with the incident wave – in fact the two polarisations ($E_\parallel$, $E_\perp$) have a different phase $\Rightarrow$ elliptically polarised light. To verify that the transmitted, evanescent, wave does in fact have zero intensity we investigate the behaviour of $\underline{S} = \underline{E}\times\underline{H}$ in the z-direction at $x = 0$. Then

p208 from p211 in notes

$$S_{zt} = E_{xt}H_{yt} = E_t\cos\theta_t e^{-kz}e^{-i\omega t}(E_t/Z_2)e^{-kz}e^{-i\omega t}$$

$$S_{zt} = E_{yt}H_{xt}$$

$$S_{zt} = -E_t e^{-kz}e^{-i\omega t}(E_t/Z_2)\cos\theta_t e^{-kz}e^{-i\omega t}$$

But $\cos\theta_t = ikc/n_2\omega$

$$S_{zt} = i[\qquad]e^{-2i\omega t} \qquad\qquad S_{zt} = i[\qquad]e^{-2i\omega t}$$

Using p208 and 211 in my notes

$$\underline{S} = \underline{E} \wedge \underline{H}$$

$$S_{zt} = E_{xt} H_{yt}$$

@ $x = 0$

$$E_{xt} = E_t \cos\theta_t e^{-kz} e^{-i\omega t}$$

$$= \frac{i E_t ck}{n_2 \omega} e^{-kz} e^{-i\omega t}$$

$$H_{yt} = \frac{E_t}{Z_2} e^{-kz} e^{-i\omega t}$$

(24-104)

$$\langle S \rangle = \tfrac{1}{2} \text{Real} \left(E_c \wedge H_c^* \right)$$

$$= \tfrac{1}{2} \text{Real} \left(\frac{i E_t^2 ck e^{-2kz}}{Z_2} \right)$$

$$= 0$$

$$S_{zt} = E_{yt} H_{xt}$$

$$E_{yt} = -E_t e^{-kz} e^{-i\omega t}$$

$$H_{xt} = \frac{E_t \cos\theta_t}{Z_2} e^{-kz} e^{-i\omega t}$$

$$= \frac{i E_t ck}{Z_2 n_2 \omega} e^{-kz} e^{-i\omega t}$$

$$\langle S \rangle = \tfrac{1}{2} \text{Real} \left(E_c \wedge H_c^* \right)$$

$$= \tfrac{1}{2} \text{Real} \left(- \frac{i E_t^2 ck}{Z_2 n_2 \omega} e^{-2kz} \right)$$

$$= 0$$

In other words in both cases ($E_{\parallel}$ and $E_{\perp}$) the time averaged energy flow in the z-direction $\langle S_{zt} \rangle$, in the second medium is purely imaginary $\Rightarrow$ Real energy flow is zero — as expected.

* Practically, what we measure is the intensity of the reflected and/or transmitted wave. Reflection and transmission coefficients are defined as

$$R = \frac{\underline{S}_r \cdot \hat{n}}{\underline{S}_i \cdot \hat{n}} \qquad T = \frac{\underline{S}_t \cdot \hat{n}}{\underline{S}_i \cdot n}$$

where $\hat{n}$ is a normal to the surface and $\underline{S}_r$, $\underline{S}_i$, $\underline{S}_t$ are the time averaged Poynting vectors (energy flow vectors)

$S = E \wedge H \quad Z = E/H \Rightarrow S = E^2/Z \quad Z = (\mu/\varepsilon)^{1/2}$

$$\underline{S} \cdot \hat{n} = \tfrac{1}{2} (\varepsilon/\mu)^{1/2} |\underline{E}|^2 \, \hat{k} \cdot \hat{n}$$

$\hat{k}$ $\rightarrow$ unit vector in wave direction

$$\Rightarrow \quad R = \left(\frac{E_r}{E_i}\right)^2$$

since $\hat{k} \cdot \hat{n} = \cos\theta_i = \cos\theta_r$ and ε, μ are same for incident + reflected waves

$$T = \left(\frac{E_t}{E_i}\right)^2 \left(\frac{\varepsilon_2 \mu_1}{\mu_2 \varepsilon_1}\right)^{1/2} \frac{\cos\theta_t}{\cos\theta_i} = \frac{Z_1}{Z_2} \frac{\cos\theta_t}{\cos\theta_i} \left(\frac{E_t}{E_i}\right)^2$$

Thus for the two possible orientations of $\underline{E}_i$ we find

$\underline{E}$ in plane of incidence

$$R = \left(\frac{Z_2 \cos\theta_t - Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i}\right)^2$$

$$T = \frac{4 Z_2 Z_1 \cos\theta_t \cos\theta_i}{(Z_2 \cos\theta_t + Z_1 \cos\theta_i)^2}$$

$\underline{E}$ @ 90° to plane of incidence

$$R = \left(\frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}\right)^2$$

$$T = \frac{Z_1}{\cancel{Z_2}} \frac{\cos\theta_t}{\cancel{\cos\theta_i}} \frac{4 Z_2^{\cancel{2}} \cos^{\cancel{2}}\theta_i}{(Z_2 \cos\theta_i + Z_1 \cos\theta_t)^2}$$

For the special case of normal incidence $(\Theta_i = 0 = \Theta_t)$ and $\mu_1 = \mu_2$ when $(Z_1/Z_2) = (\varepsilon_2/\varepsilon_1)^{1/2} = n_2/n_1$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad = \quad R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2} \quad = \quad T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

plane of incidence is defined by normal to interface and $\underline{k}$ etc.

equal, since at normal incidence there is no plane of incidence and hence no distinction between parallel and @ 90° to the plane

Where of course $R + T = 1$ (energy conservation)

e.g. Water $n_2 = 4/3$ air $n_1 = 1$

then $R = \left(\frac{1 \times 3}{3 \times 7}\right)^2 = 0.02$

$\Downarrow$

2% of incident energy is reflected.

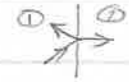
Include the following material only when waves in conductors have been discussed.

Assign Chap 25 Non Cond problems → Go to Ch 24 on conductors p 198 in notes

* <u>Reflection from the surface of a conductor</u> Ch 25 : 1, 3, 5, 7, 9

Consider only the special case; $\mu_1 = \mu_2$ — non magnetic media normal incidence $\Theta_i = \Theta_t = \Theta_r = 0$ $[Z_1/Z_2 = n_2/n_1]$

[Medium ② is the conductor – Metal]



Then for either orientation of $\underline{E}$ ($E_\perp$ and $E_\parallel$) we find that

$$(E_r/E_i) = \frac{n_1 - n_2}{n_1 + n_2}$$

$$(E_t/E_i) = \frac{2n_1}{n_1 + n_2}$$

Remember that one method of dealing with waves in a