For the special case of normal incidence $(\Theta_i = 0 = \Theta_t)$ and $\mu_1 = \mu_2$ when $(Z_1/Z_2) = (\varepsilon_2/\varepsilon_1)^{1/2} = n_2/n_1$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad = \quad R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \quad = \quad T = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

plane of incidence is defined by normal to interface and $\underline{k}$ inc.

equal, since at normal incidence there is no plane of incidence and hence no distinction between parallel and @ 90° to the plane

Where of course $R + T = 1$ (energy conservation)

e.g. Water $n_2 = 4/3$ air $n_1 = 1$

then $R = \left(\frac{1 \times 3}{3 \times 7}\right)^2 = 0.02$

$\Downarrow$

2% of incident energy is reflected.

Include the following material only when waves in conductors have been discussed.

Assign Chap 25 Non Cond problems → Go to Ch 24 on conductors p 198 in notes

* Reflection from the surface of a conductor

Ch 25: 1, 3, 5, 7, 9

Consider only the special case; $\mu_1 = \mu_2$ – non magnetic media

normal incidence $\Theta_i = \Theta_t = \Theta_r = 0$ $[Z_1/Z_2 = n_2/n_1]$

[Medium ② is the conductor – metal]

Then for either orientation of $\underline{E}$ ($E_\perp$ and $E_\parallel$) we find that

$$(E_r/E_i) = \frac{n_1 - n_2}{n_1 + n_2}$$

$$(E_t/E_i) = \frac{2 n_1}{n_1 + n_2}$$

Remember that one method of dealing with waves in a conductor was to use a complex index of refraction

$$n = n' + i\left(\frac{c\beta}{\omega}\right)$$

where $\beta = \omega\left(\frac{\mu\varepsilon}{2}\right)^{1/2}\left[\left(1+\frac{1}{Q^2}\right)^{1/2} - 1\right]^{1/2}$ $\qquad Q = \frac{\omega\varepsilon}{\sigma}$

and $n' = \frac{c\alpha}{\omega} = \frac{c}{v}$ and $\alpha = \omega\left(\frac{\mu\varepsilon}{2}\right)^{1/2}\left[\left(1+\frac{1}{Q^2}\right)^{1/2} + 1\right]^{1/2}$

In this case we obtain

$$(E_r/E_i) = \frac{n_1 - n_2' - i(c\beta_2/\omega)}{n_1 + n_2' + i(c\beta_2/\omega)}$$

$$(E_t/E_i) = \frac{2n_1}{n_1 + n_2' + i(c\beta_2/\omega)}$$

Since (E_r/E_i) and (E_t/E_i) are complex we immediately conclude that the reflected and transmitted waves are phase shifted w.r.t the incident wave $[E_r = E_i A e^{i\phi}]$

Also, for good conductors $\sigma_2 \to \infty$, $Q_2 \to 0$, $\beta_2 \to \infty$

we see that $\left(\frac{E_r}{E_i}\right) \to -1$ $\quad \left(\frac{E_t}{E_i}\right) \to$ purely imaginary

In terms of R and T

$$R \to 1 \quad \text{and} \quad T \to 0$$

i.e. a metal reflects all the incident energy.

N.B. This assumption is based upon $Q \to 0$ since $\sigma \to \infty$

But if ω is large, even for large σ Q will not be zero. In such cases we can have significant transmission.

e.g. Cu, Au reflect most of low frequency light and hence appear yellowish.

Au reflecting red-yellow is a property of Au — nothing to do with attenuation depth. Thus if white light is incident on Au the red-yellow is reflected, leaving the green-blue to be transmitted. Of course δ must be small enough (the foil thin enough) to allow any transmission.

If a thin enough sheet is observed in transmission only high frequency — Green + Blue — light will be observed.

End of discussion of reflection from the surface of a conductor

* Radiation Pressure (stat Griffiths p359)

Finally, in this chapter, we consider the concept of radiation pressure.

$F = q(\underline{E} + \underline{v} \wedge \underline{B})$

$\underline{J}_f = \rho \underline{v}$

Consider a volume V containing charges and currents.

$F = q\left(\underline{E} + \frac{\underline{J} \wedge \underline{B}}{\rho}\right)$

Then

$$\underline{F} = \int_V \left[\rho_f \underline{E} + \left(\underline{J}_f \wedge \underline{B} \right) \right] d\tau = \int \underline{f}_v \, d\tau$$

$dF = \rho\left(E + \frac{J \wedge B}{\rho}\right) d\tau = \left(\rho \underline{E} + \underline{J} \wedge \underline{B}\right)$

= total force on charges

Using Maxwells equations with $\underline{P}$ and $\underline{M} = 0$

$\rho_f = \varepsilon_0 (\underline{\nabla} \cdot E)$

$\underline{J}_f = \frac{1}{\mu_0}\left[\underline{\nabla} \wedge B - \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right]$

$$\underline{f}_v = \varepsilon_0 (\underline{\nabla} \cdot \underline{E}) \underline{E} + \frac{1}{\mu_0} (\underline{\nabla} \wedge \underline{B}) \wedge \underline{B} - \varepsilon_0 \frac{\partial \underline{E}}{\partial t} \wedge \underline{B}$$

but

$$\frac{\partial \underline{E}}{\partial t} \wedge \underline{B} = \frac{\partial}{\partial t} (\underline{E} \wedge \underline{B}) - \underline{E} \wedge \frac{\partial \underline{B}}{\partial t}$$

$$= \frac{\partial}{\partial t} (\underline{E} \wedge \underline{B}) + \underline{E} \wedge (\underline{\nabla} \wedge \underline{E})$$

$$\Rightarrow \underline{f}_v = \varepsilon_0 \left[(\underline{\nabla} \cdot \underline{E}) \underline{E} - \underline{E} \wedge (\underline{\nabla} \wedge \underline{E}) \right] + \frac{1}{\mu_0} \left[\underbrace{(\underline{\nabla} \cdot \underline{B}) \underline{B}}_{\left[\text{zero } \underline{\nabla} \cdot \underline{B} = 0\right]} - \underline{B} \wedge (\underline{\nabla} \wedge \underline{B}) \right]$$

$$- \varepsilon_0 \frac{\partial}{\partial t} (\underline{E} \wedge \underline{B})$$

Using $\underline{\nabla}(B^2) = 2 \underline{B} \wedge (\underline{\nabla} \wedge \underline{B}) + 2 (\underline{B} \cdot \underline{\nabla}) \underline{B}$ [Vector identity of $\underline{\nabla}(\underline{A} \cdot \underline{B})$]

and the same for $\underline{\nabla}(E^2)$

we obtain

$$f_v = \varepsilon_0\left[(\underline{\nabla}\cdot\underline{E})\underline{E} + (\underline{E}\cdot\underline{\nabla})\underline{E}\right] + \frac{1}{\mu_0}\left[(\underline{\nabla}\cdot\underline{B})\underline{B} + (\underline{B}\cdot\underline{\nabla})\underline{B}\right]$$
$$- \tfrac{1}{2}\underline{\nabla}\left[\varepsilon_0 E^2 + \tfrac{1}{\mu_0}B^2\right] - \varepsilon_0\mu_0\frac{\partial}{\partial t}(\underline{E}\wedge\underline{H})$$

the first 3 terms can be combined by introducing the <u>Maxwell Stress Tensor</u> T with elements given by

$$T_{ij} = \varepsilon_0\left(E_iE_j - \tfrac{1}{2}E^2\delta_{ij}\right) + \tfrac{1}{\mu_0}\left(B_iB_j - \tfrac{1}{2}B^2\delta_{ij}\right)$$

$$\left[\; i,j \text{ are over } x,y,z \text{ and } \delta_{ij} = \begin{cases} 0 & (i\neq j) \\ 1 & (i=j)\end{cases} \;\right]$$

so that

$$\left[f_v + \mu_0\varepsilon_0\frac{\partial}{\partial t}[\underline{S}]\right]_x = \left[\underline{\nabla}\cdot\underline{T}\right]_x = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z}$$

$$\left[\text{where } (\underline{\nabla}\cdot\underline{T})_j = \sum_{i=xyz}\frac{\partial}{\partial\xi_i}T_{ji} \quad \text{and } \xi_i = x \;\; \xi_j = y \;\; \xi_k = z\right]$$

The Maxwell stress tensor is not at issue here. We now extend our volume to all space then

$$\underline{F}_x = \left[\int_{\text{all space}} f_v \, d\tau\right]_x = \left[\int_{\text{all space}} -\mu_0\varepsilon_0\frac{\partial}{\partial t}[\underline{S}]\, d\tau\right]_x + \underbrace{\int_{\text{all space}}[\underline{\nabla}\cdot\underline{T}]_x\, d\tau}_{\Leftarrow\ \oint_S \underline{T}_x\cdot d\underline{a}}$$

actually T is a tensor but considering components can show by applying divergence theorem

$$T_x = T_{xx}\hat{x} + T_{xy}\hat{y} + T_{xz}\hat{z}$$

but $da \propto R^2$
and $T \propto E^2$ (or B^2)
$\propto \left(\frac{1}{R^2}\right)^2$

$\Rightarrow$ as $R\to\infty$ (all space) the surface integral $\Rightarrow 0$

N.B. Remember $\oint \underline{S}\cdot d\underline{a}$ is energy flow

$$\Rightarrow \qquad \underline{F} + \mu_0 \varepsilon \frac{d}{dt}\left[\int S \, d\tau\right] = 0$$

Interpreting $\underline{F} = dp/dt$ rate of change of momentum of particles

then $$\frac{d}{dt}\left[p_{matter} + \mu_0 \varepsilon_0 \int \underline{S} \, d\tau \right] = 0$$

When the momentum of the particles change, to conserve momentum the EM field is assumed to "take-up" the remaining momentum, thus we interpret

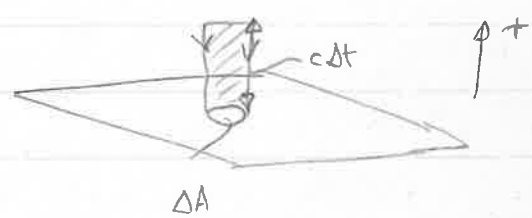
$$\mu_0 \varepsilon \int \underline{S} \, d\tau = p_{EMfield}$$

and <u>Momentum density</u>

$$g = \mu_0 \varepsilon_0 \underline{S}$$

$$g = \frac{S}{c^2}$$

* The normal component of g is of interest in defining a pressure. For a wave of normal incidence on area ΔA we have



$$g_i \, \Delta\tau = \frac{S_i \Delta\tau}{c^2}$$

$$\underline{P_i} = \frac{\underline{S_i} \Delta A \; c\Delta t}{c^2} \qquad \text{(momentum of incident wave in volume } \Delta\tau\text{)}$$

With a similar expression for the reflected and transmitted waves

$$\Rightarrow \quad \Delta P = \underline{P_r} - \underline{P_i} = (\underline{S_r} - \underline{S_i}) \frac{\Delta A \, \Delta t}{c}$$

Assuming $\underline{P}_r$ is + then $\underline{P}_r > 0$ $\underline{P}_i < 0$

$$|\Delta \underline{P}| = \left[|\underline{S}_r| + |\underline{S}_i|\right] \frac{\Delta A \, \Delta t}{c}$$

$$\Rightarrow \text{Pressure} = \frac{F}{\Delta A} = \frac{\Delta P}{\Delta t \, \Delta A} = \left[|\underline{S}_r| + |\underline{S}_i|\right]/c$$

now $|\underline{S}_r| = R|\underline{S}_i|$

$\Rightarrow$ Pressure due to radiation, at normal incidence, is given by

$$\frac{(1+R)|\underline{S}_i|}{c}$$

or, since $|\underline{S}_i| = u_i c$ [c - speed of light
u - energy density]

$$\text{pressure} = (1+R)\, u_i$$

[For non normal incidence we can write
$\text{pressure} = u_i(1+R)\cos^2\theta_i$]

* Note that, as expected, if $R \sim 1$ (e.g. good conductors), then the pressure is $2u_i$, Twice the value of that on a good absorber ($R \approx 0$).

Also, we have implicitly assumed in the above analysis that u_i, $\underline{S}_i$ are the time averaged values. This is what is actually measured in nearly all cases since the frequency is large.

Problems : Chapter 25 : 1, 3, 5, 7, 9 non-conducting media

11, 12, 13 conducting media

16, 17 radiation pressure

need

[solar constant $\langle S \rangle = 1340$ watts/m^2

Mass of sun $= 2 \times 10^{30}$ kg

$G = 6.67 \times 10^{-11}$ $N \cdot m^2 / kg^2$]