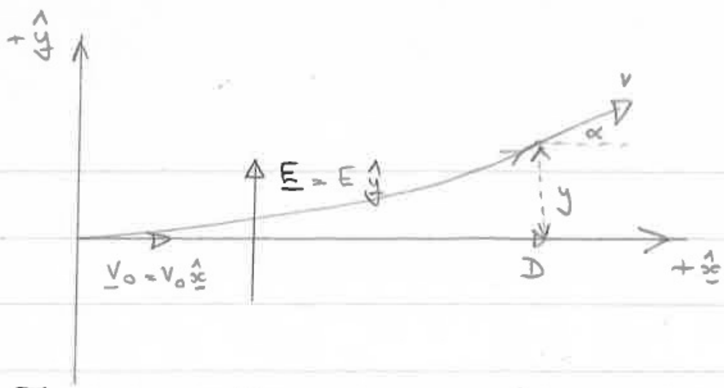
A-1)



$$\underline{F} = q\underline{E} = qE\hat{y} = M\underline{a}$$

$$\Rightarrow \quad M a_x = 0 \quad \Rightarrow \quad V_x = V_0 \quad \Rightarrow x = V_0 t$$

$$M a_y = qE$$

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$$M \frac{dv_y}{dt} = qE \qquad V_y = (q/M) E t$$

$$\frac{dy}{dt} = \frac{q}{M} E t$$

+1

$$\Rightarrow \quad y = \frac{q}{2M} E t^2 = \frac{qEt^2}{2M}$$

(a) Equation of its path is given by

2

$$y = \frac{qEt^2}{2M} = \frac{qE}{2M}\frac{x^2}{V_0^2}$$

1

(b) when $x = D$ $\quad y = d = \dfrac{qED^2}{2MV_0^2}$

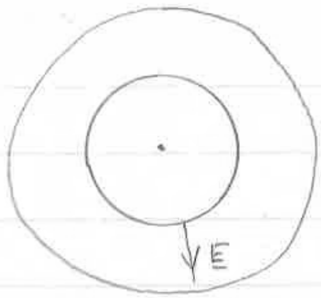
(c) At (D, d) $\quad V_x = V_0 \qquad t = D/V_0$

2

$$\Rightarrow \quad V_y = \frac{qE}{M}\frac{D}{V_0}$$

$$\Rightarrow \tan\alpha = \frac{V_y}{V_x} = \frac{qED}{MV_0^2}$$

A-3)



$\underline{E}$ is radial
$\underline{E} = E\hat{\rho}$

For circular orbits the centripetal acceleration must be given by v_ϕ^2/ρ

$$\Rightarrow \quad qE = \frac{mv_\phi^2}{\rho}$$

$$\text{KE in orbit} = \tfrac{1}{2}mv_\phi^2$$

For a system of co-axial cylinders, as above

$$E = \frac{Q}{2\pi\rho l \varepsilon_0}$$ (for length l) [Using Gauss Law]

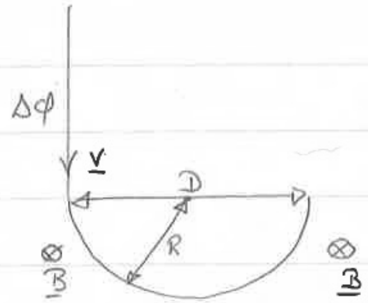
$$\Delta\phi = \frac{Q}{2\pi l \varepsilon_0}\int_a^b \frac{d\rho}{\rho} = \frac{Q}{2\pi l \varepsilon_0}\ln(b/a)$$

$$\Rightarrow \quad E = \frac{\Delta\phi}{\rho(\ln b/a)}$$

$$\therefore \quad qE = \frac{\Delta\phi\, q}{\cancel{\rho}(\ln b/a)} = \frac{mv_\phi^2}{\cancel{\rho}}$$

$$\therefore \quad KE = \tfrac{1}{2}mv_\phi^2 = \frac{\tfrac{1}{2}\Delta\phi\, q}{\ln(b/a)}$$

A-5)



Production at rest implies that $\frac{1}{2}Mv^2 = q\Delta\phi$

For motion in magnetic field

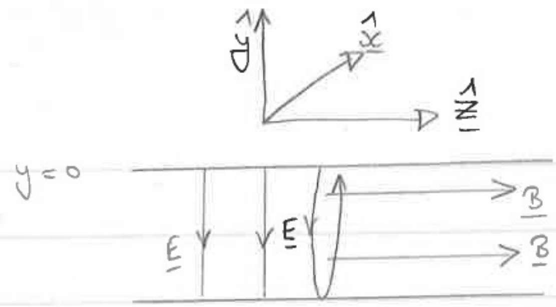
$$\frac{Mv^2}{R} = qvB$$

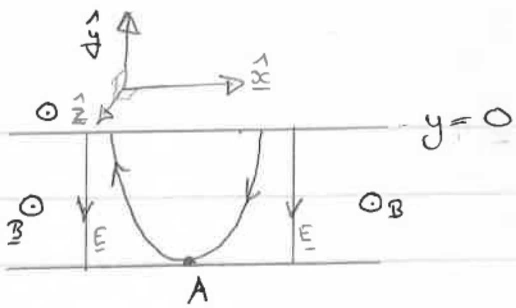
$$v = \frac{RqB}{M}$$

$$\Rightarrow \quad \frac{\cancel{M}}{2}\frac{R^2 q^2 B^2}{M^2} = \cancel{q}\Delta\phi$$

$$M = \frac{D^2 q B^2}{4.2\Delta\phi} = \frac{qB^2D^2}{8\Delta\phi}$$

A-9)





$\underline{E} = -E\hat{y} \quad \underline{B} = B\hat{z}$

Motion of particle is restricted to xy plane
Using energy conservation ($\underline{B}$ does no work on particle)

$$\tfrac{1}{2}m(v_x^2 + v_y^2) + q\phi(y) = \text{constant}$$

At $t = 0$ $\quad v_x = v_y = 0 \quad$ and $\quad \phi = \phi(0)$

$\Rightarrow$ the constant on r.h.s above is $q\phi(0)$

$$\therefore \quad \tfrac{1}{2}m(v_x^2 + v_y^2) = q(\phi(0) - \phi(y))$$

At point A above the particle has only an x component of velocity $v_x(A)$

$$\Rightarrow \quad \tfrac{1}{2}m(v_x(A))^2 = q\Delta\phi \qquad \text{—— (1)}$$

The equation of motion of the charged particle is given by

$$\underline{F} = m\underline{a} = q(\underline{E} + \underline{v}\wedge\underline{B}) = q(-E\hat{y} + \underline{v}\wedge\underline{B})$$

$$\underline{v}\wedge\underline{B} = (v_x\hat{x} + v_y\hat{y})\wedge B\hat{z}$$
$$= -v_xB\hat{y} + v_yB\hat{x}$$

$$\Rightarrow \quad m\underline{a} = \hat{x}\,qv_yB - \hat{y}\,q(E + v_xB)$$

but $\underline{v} = v_x\hat{x} + v_y\hat{y}$

$$\Rightarrow \quad \underline{a} = \frac{d\underline{v}}{dt} = \hat{x}\frac{dv_x}{dt} + \hat{y}\frac{dv_y}{dt}$$

Equating x components

$$M\frac{dv_x}{dt} = qv_yB = qB\frac{dy}{dt}$$

$$\Rightarrow \int M\,dv_x = \int qB\,dy$$

$$V_x = \frac{qB}{M}y + \text{constant}$$

When $y = 0$ $V_x = 0$ $\Rightarrow$ constant is zero

$\therefore$ @ A $y = -d$

$$\Rightarrow V_x(A) = -\frac{qBd}{M}$$

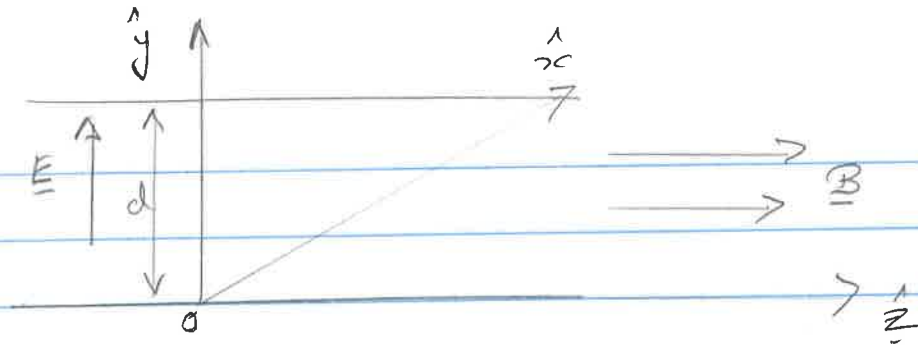
Substituting in (i) above we have

$$\frac{M}{2}\frac{q^2B^2d^2}{M^2} = q\Delta\phi$$

$$\Rightarrow \Delta\phi = \frac{qB^2d^2}{2M}$$

If $\Delta\phi$ is larger than this value the particle will not "turn-over" before reaching the lower plate and a current will be detected between the plates. Thus for no current $\Delta\phi$ must be less than the value given above.

A-9) alt)



$$\underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

$$\underline{v} \wedge \underline{B} = \hat{\underline{x}} B v_y - \hat{y} v_x B$$

$$\underline{F} = \hat{\underline{x}} q B v_y + \hat{y}(qE - qv_x B)$$

$$M\frac{dV_x}{dt} = qBv_y \quad : \quad M\frac{dv_y}{dt} = qE - BV_x q$$

[No motion along z]

x component $\Rightarrow$ $\frac{dV_y}{dt} = \frac{M}{qB}\frac{d^2 v_x}{dt^2}$

Substituting in y equation

$$\frac{M^2}{qB}\frac{d^2 v_x}{dt^2} = -Bq\,V_x + qE$$

$$\frac{d^2 v_x}{dt^2} = -\frac{B^2 q^2}{M^2}V_x + \frac{q^2 EB}{M^2}$$

solution is $V_x = V_{ox}\cos(\omega t + \phi) + A$

then $\frac{dv_x}{dt} = -\omega V_{ox}\sin(\omega t + \phi)$

$$\frac{d^2 v_x}{dt^2} = -\omega^2 V_{ox}\cos(\omega t + \phi)$$

$$\Rightarrow \quad -\omega^2 V_{ox}\cos(\omega t + \phi) = -\frac{B^2 q^2}{M^2}V_{ox}\cos(\omega t + \phi)$$

$$-\frac{AB^2 q^2}{M^2} + \frac{q^2 EB}{M^2}$$

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(alt)

$\Rightarrow \quad \omega = Bq/m \quad$ and $\quad A = E/B$

Therefore $\quad v_x = v_{0x}\cos(\omega t + \phi) + E/B \qquad - (1)$

and $\quad dv_x/dt = -\omega v_{0x}\sin(\omega t + \phi)$

Using y eqn of motion

$$-m\omega v_{0x}\sin(\omega t + \phi) = qBv_y$$

$$v_y = -v_{0x}\sin(\omega t + \phi) \qquad - (2)$$

Now @ $t = 0 \quad v_x = v_y = 0$

From (2) $\quad \phi = 0 \quad (\text{or } n\pi)$

From (1) $\quad v_{0x} = -E/B$

$\Rightarrow \quad v_x = -E/B \cos\omega t + E/B \qquad - (3)$

$\qquad v_y = E/B \sin\omega t \qquad - (4)$

Now for trajectories integrate (3) + (4)

$$x = -\frac{E}{B\omega}\sin\omega t + \frac{Et}{B} + C$$

$$y = -\frac{E}{B\omega}\cos\omega t + D$$

assuming @ $t = 0 \quad x = y = 0$

then $\quad C = 0$

$$D = \frac{E}{B\omega}$$

A9-3
(A11)

$$\Rightarrow x = -\frac{E}{B\omega}\sin\omega t + \frac{Et}{B}$$

$$y = -\frac{E}{B\omega}(\cos\omega t - 1)$$

Now when $v_y = 0$ from (4) $\sin\omega t = 0$

$$\omega t = n\pi$$

so that $y = -\frac{E}{B\omega}(\cos n\pi - 1)$

$$= -\frac{E}{B\omega}\left((-1)^n - 1\right)$$

~~If $y = d$ when~~

$\Rightarrow$ for $n = 0, 2, 4$ $y = 0$

& for odd n $y = d$

the

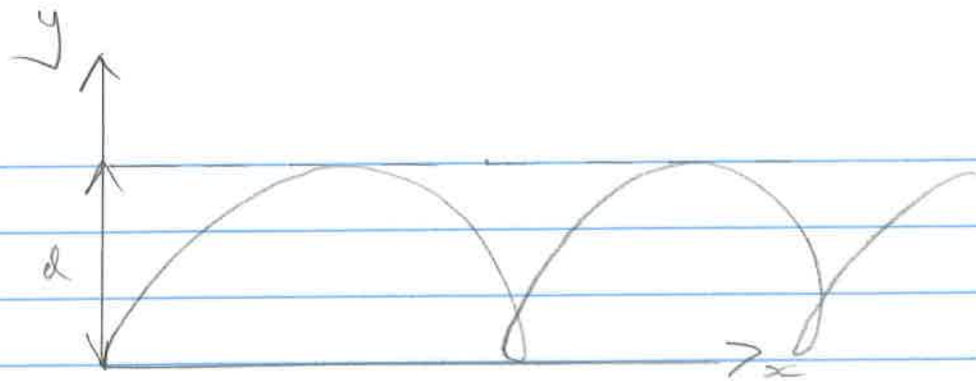
$$d = \frac{2E}{B\omega}$$

where $E = \Delta\phi / d$ and $\omega = Bq/m$

$$\Rightarrow d = \frac{2\Delta\phi m}{dB^2 q}$$

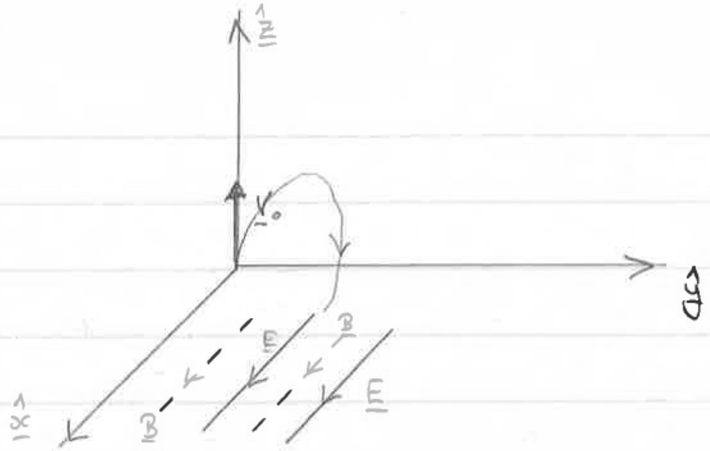
$$\Delta\phi = \frac{B^2 d^2 q}{2m}$$

Under the influence of the $\underline{E}$ and $\underline{B}$ fields the particle develops a constant (drift) velocity in the $\hat{x}$ direction whilst oscillating in $\hat{y}$



example of cycloid type of motion

A-11)



$\underline{V}_0 = v_0 \hat{\underline{z}}$

$\underline{B} = B \hat{\underline{x}}$

$\underline{E} = E \hat{\underline{x}}$

$$\underline{F} = q\left(\underline{E} + \underline{v} \wedge \underline{B}\right) \qquad \underline{v} = \hat{\underline{x}} v_x + \hat{\underline{y}} v_y + \hat{\underline{z}} v_z$$

$$m\underline{a} = q\left(E\hat{\underline{x}} + \hat{\underline{y}} B v_z - \hat{\underline{z}} B v_y\right)$$

$$\Rightarrow m\frac{dv_x}{dt} = qE \; ; \; m\frac{dv_y}{dt} = qBv_z \; ; \; m\frac{dv_z}{dt} = -qBv_y$$

$$m\frac{d^2v_y}{dt^2} = qB\frac{dv_z}{dt} \; ; \; \frac{dv_z}{dt} = -\frac{qB}{m}v_y$$

$$\Rightarrow m\frac{d^2v_y}{dt^2} = -\frac{q^2B^2}{m}v_y$$

$$\frac{d^2v_y}{dt^2} = -\frac{q^2B^2}{m^2}v_y$$

$$\Rightarrow v_y = \alpha\cos\left(\frac{qB}{m}t + \beta\right)$$

When $t = 0$ $v_y = 0$ $\Rightarrow \alpha\cos\beta = 0$ $\beta = \pi/2$

$t = 0$ $dv_y/dt = a_y = \frac{qv_0B}{m}$

$$\frac{dv_y}{dt} = -\alpha\frac{qB}{m}\sin\left(\frac{qBt}{m} + \beta\right)$$

$$\Rightarrow \frac{\cancel{q}v_0\cancel{B}}{\cancel{m}} = -\frac{\alpha\cancel{q}\cancel{B}}{\cancel{m}}\sin\beta$$

$$\alpha = -v_0$$

$$\therefore \quad V_y = -V_0 \cos\left(\frac{qB}{m}t + \frac{\pi}{2}\right) \qquad \text{—} \qquad (1)$$

$$\frac{dv_y}{dt} = \frac{V_0 qB}{m} \sin\left(\frac{qB}{m}t + \pi/2\right) = \frac{qB}{m} V_z$$

$$\Rightarrow \quad V_z = V_0 \sin\left(\frac{qBt}{m} + \pi/2\right)$$

[which satisfies the condition @ $t=0$ $V_z = V_0$]

When particle reaches its maximum value in z

$$\frac{dz}{dt} = 0 = V_z$$

$$\Rightarrow \quad 0 = \sin\left(\frac{qB}{m}t + \pi/2\right)$$

$$\frac{qBt}{m} + \frac{\pi}{2} = n\pi \quad (n = 0, 1, 2 \ldots$$

[first maximum value occurs at $n = 1$]

$$\Rightarrow \quad t = \frac{\pi}{2}\frac{m}{qB}$$

$$\frac{dz}{dt} = V_0 \sin\left(\frac{qBt}{m} + \frac{\pi}{2}\right)$$

$$z = -\frac{mV_0}{qB}\cos\left(\frac{qBt}{m} + \frac{\pi}{2}\right) + A_1$$

when $t=0$ $z=0$ $\Rightarrow A=0$

then $$Z_{MAX} = -\frac{mV_0}{qB}\cos\pi = \frac{mV_0}{qB}$$

When $z = \frac{1}{2}Z_{MAX}$ $$\frac{mV_0}{2qB} = -\frac{mV_0}{qB}\cos\left(\frac{qBt}{m} + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \frac{qBt}{m} + \frac{\pi}{2} = \cos^{-1}\left(-\tfrac{1}{2}\right) = \frac{2\pi}{3}$$

$$t_{1/2} = \frac{\pi}{6}\frac{m}{qB}$$

From (1) $\dfrac{dy}{dt} = -V_0 \cos\left(\dfrac{qBt}{M} + \dfrac{\pi}{2}\right)$

$$\Rightarrow \quad y = -\frac{V_0 M}{qB}\sin\left(\frac{qBt}{M} + \frac{\pi}{2}\right) + \mathcal{B}$$

@ $t = 0 \quad y = 0 \quad \Rightarrow \quad \mathcal{B} = MV_0/qB$

When $t = t_{1/2} = \dfrac{M\pi}{6qB}$

$$y = -\frac{MV_0}{qB}\sin\left(\pi/6 + \pi/2\right) + \frac{MV_0}{qB}$$

$$= \frac{MV_0}{qB}\left(1 - \frac{\sqrt{3}}{2}\right) = \underline{\underline{0.134\,\frac{MV_0}{qB}}}$$

For x motion we have $M\dfrac{dv_x}{dt} = qE$

$$V_x = \frac{q}{M}Et + c$$

When $t = 0 \quad V_x = 0 \quad \Rightarrow \quad c = 0$

$$\frac{dx}{dt} = \frac{qEt}{M}$$

$$x = \frac{qE}{M}\int t\,dt = \frac{qEt^2}{2M} + D$$

When $t = 0 \quad x = 0 \quad \Rightarrow \quad D = 0$

$\therefore$ When $t = t_{1/2} = \dfrac{M\pi}{6qB}$

$$x = \frac{qEM^2\pi^2}{2M\,36q^2B^2} = \underline{\underline{\frac{EM\pi^2}{72qB^2}}}$$