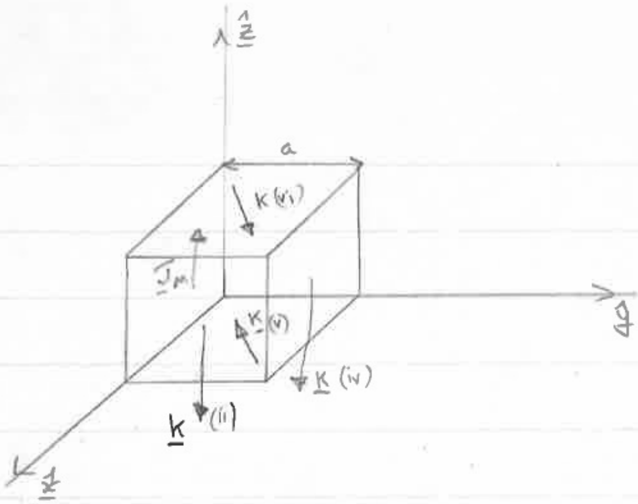
20-3)



$$\underline{M} = -(M/a)y\,\hat{\underline{x}} + (M/a)x\,\hat{\underline{y}}$$

(a) $$\underline{J}_M = \underline{\nabla}\wedge\underline{M} = \begin{vmatrix} \hat{\underline{x}} & \hat{\underline{y}} & \hat{\underline{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\frac{M}{a}y & \frac{M}{a}x & 0 \end{vmatrix}$$

$$= \hat{\underline{z}}\left(\frac{M}{a} + \frac{M}{a}\right)$$

$$= \frac{2M}{a}\,\hat{\underline{z}}$$

(b) $\underline{K}_M = \underline{M}\wedge\hat{\underline{n}}$ where $\hat{\underline{n}}$ is an outgoing normal.

$\hat{\underline{n}}$ has different direction on each of six sides

(i) $x = 0$ $\quad \hat{\underline{n}} = -\hat{\underline{x}} \Rightarrow \underline{K}_M = \hat{\underline{z}}\,M/a\,x = 0$

(ii) $x = a$ $\quad \hat{\underline{n}} = \hat{\underline{x}} \Rightarrow \underline{K}_M = -\hat{\underline{z}}\,M/a\,x = -M\hat{\underline{z}}$

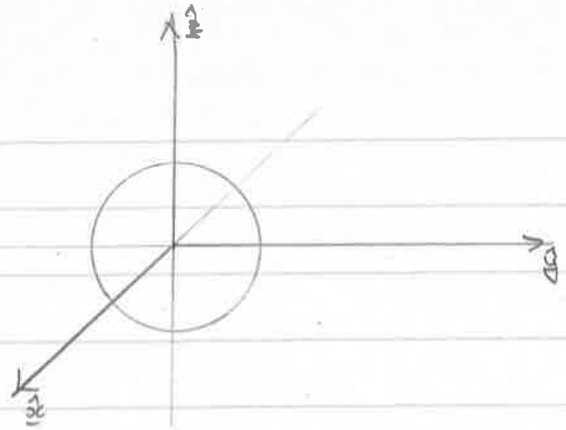
(iii) $y = 0$ $\quad \hat{\underline{n}} = -\hat{\underline{y}} \Rightarrow \underline{K}_M = \hat{\underline{z}}\frac{M}{a}y = 0$

(iv) $y = a$ $\quad \hat{\underline{n}} = \hat{\underline{y}} \Rightarrow \underline{K}_M = -\hat{\underline{z}}\,M/a\,y = -M\hat{\underline{z}}$

(v) $z = 0$ $\quad \hat{\underline{n}} = -\hat{\underline{z}} \Rightarrow \underline{K}_M = -\hat{\underline{y}}\frac{M}{a}y - \hat{\underline{x}}\frac{M}{a}x$

(vi) $z = a$ $\quad \hat{\underline{n}} = \hat{\underline{z}} \Rightarrow \underline{K}_M = M/a\,(\hat{\underline{x}}x + \hat{\underline{y}}y)$

20-5)



$$\underline{M} = (\alpha z^2 + \beta)\hat{\underline{z}}$$

α

(a) α units are M/z^2 = $\frac{\text{ampere}}{m^3}$

β " " M = ampere/m

(b)
$$\underline{J}_M = \nabla \wedge \underline{M}$$
$$= \hat{\underline{r}} \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta M_\phi) - \frac{\partial M_\theta}{\partial\phi}\right]$$
$$+ \hat{\underline{\theta}} \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial M_r}{\partial\phi} - \frac{\partial(M_\phi r)}{\partial r}\right]$$
$$+ \hat{\underline{\phi}} \frac{1}{r}\left[\frac{\partial}{\partial r}(r M_\theta) - \frac{\partial M_r}{\partial\theta}\right]$$

where
$$\underline{M} = (\alpha z^2 + \beta)\hat{\underline{z}}$$
$$= (\alpha r^2\cos^2\theta + \beta)\left[\hat{\underline{r}}\cos\theta - \hat{\underline{\theta}}\sin\theta\right]$$
$$\Rightarrow M_r = (\alpha r^2\cos^2\theta + \beta)\cos\theta$$
$$M_\theta = -(\alpha r^2\cos^2\theta + \beta)\sin\theta$$
$$M_\phi = 0$$

Also $\frac{\partial M_\theta}{\partial\phi} = 0$ $\frac{\partial M_r}{\partial\phi} = 0$ leaving only the $\hat{\phi}$ term in $\underline{J}_M$ above

$$\underline{J}_M = \hat{\underline{\phi}} \frac{1}{r}\left[M_\theta + r\frac{\partial M_\theta}{\partial r} - \frac{\partial M_r}{\partial\theta}\right]$$

$$\frac{\partial M_\theta}{\partial r} = -2\alpha r\cos^2\theta\sin\theta \; ; \; \frac{\partial M_r}{\partial\theta} = -\beta\sin\theta - 3\alpha r^2\cos^2\theta\sin\theta$$

$$\Rightarrow \underline{J}_M = \hat{\phi}\,\frac{1}{r}\Big[-\alpha r^2\cos^2\theta\sin\theta - \beta\sin\theta - 2\alpha r^2\cos^2\theta\sin\theta + \beta\sin\theta + 3\alpha r^2\cos^2\theta\sin\theta\Big]$$

$$= 0$$

[Or in rectangular co-ordinates $\nabla \wedge \underline{M}$ is clearly zero]

$$\underline{J}_M = 0$$

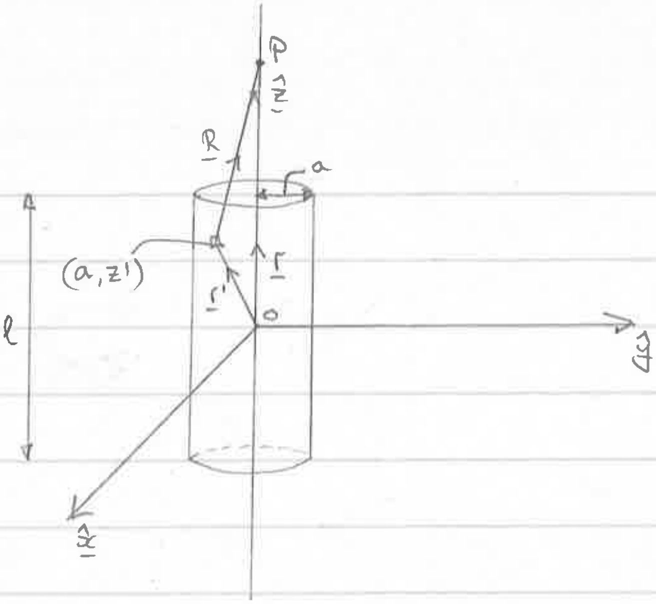
(c) $\underline{K}_M = \underline{M} \wedge \hat{\underline{n}}$ where $\hat{\underline{n}} = \hat{\underline{r}}$ (at $r = a$)

4 Using $\underline{M} = (\alpha r^2\cos^2\theta + \beta)(\hat{\underline{r}}\cos\theta - \hat{\underline{\theta}}\sin\theta)$

$$\underline{M} \wedge \hat{\underline{n}} = \hat{\underline{\phi}}\sin\theta(\alpha r^2\cos^2\theta + \beta) \quad \text{at } r = a$$

$$= \hat{\underline{\phi}}\sin\theta(\alpha a^2\cos^2\theta + \beta)$$

20-7)



$$\underline{M} = M\hat{\underline{z}}$$

(a) $\underline{J}_M = \nabla \wedge \underline{M} = 0$

$\underline{k}_M = \underline{M} \wedge \hat{\underline{n}} = M\,\hat{\underline{z}} \wedge \hat{\underline{n}}$

For cylinder ends $\hat{\underline{n}} = \pm\hat{\underline{z}} \Rightarrow \underline{k}_M = 0$ on ends

For curved surface $\hat{\underline{n}} = \hat{\underline{\rho}}$

$\Rightarrow \underline{k}_M = M\hat{\underline{z}} \wedge \hat{\underline{\rho}} = \hat{\underline{\phi}}\,M$

We can now find $\underline{B}$ from $\underline{B} = \dfrac{\mu_0}{4\pi}\oint_{S'} \dfrac{(\underline{k}_M \wedge \hat{\underline{R}})}{R^2}\,da'$

[Since $\underline{J}_M = 0$ there is no volume current density term]

For P on z axis $\underline{R} = \underline{r} - \underline{r}'$ where $\underline{r} = z\hat{\underline{z}}$

and $\underline{r}' = a\hat{\underline{\rho}} + z'\underline{z}'$

$\underline{R} = \hat{\underline{z}}(z - z') - a\hat{\underline{\rho}}$

$R = \left[(z-z')^2 + a^2\right]^{1/2}$

and $da' = a\,d\phi'\,dz'$

$$\underline{k}_M \wedge \underline{R} = M\hat{\underline{\phi}} \wedge \left[\hat{\underline{z}}\,(z-z') - a\hat{\underline{\rho}}\right]$$
$$= M(z-z')\,\hat{\underline{\rho}} + \hat{\underline{z}}\,aM$$
$$= M(z-z')\left[\hat{\underline{x}}\cos\phi + \hat{\underline{y}}\sin\phi\right] + aM\hat{\underline{z}}$$

Thus $\underline{B} = \dfrac{\mu_0}{4\pi}\displaystyle\int \dfrac{\left[M(z-z')\left(\hat{\underline{x}}\cos\phi + \hat{\underline{y}}\sin\phi\right) + aM\hat{\underline{z}}\right] a\,d\phi'\,dz'}{\left[(z-z')^2 + a^2\right]^{3/2}}$

Integral over ϕ' is $0 \to 2\pi$ and $\int_0^{2\pi} \sin\phi \, d\phi = 0 = \int_0^{2\pi} \cos\phi \, d\phi$

$\Rightarrow$ $\hat{x}$ and $\hat{y}$ components of $\underline{B}$ are zero

$$\Rightarrow \underline{B} = \hat{\underline{z}} \frac{\mu_0 M a^2}{4\pi} \int \frac{d\phi' \, dz'}{\left[(z-z')^2 + a^2\right]^{3/2}}$$

$$= \hat{\underline{z}} \frac{\mu_0 M a^2 \cancel{2\pi}}{2 \cancel{4\pi}} \int_{-l/2}^{+l/2} \frac{dz'}{\left[(z-z')^2 + a^2\right]^{3/2}}$$

Try $z - z' = a \tan\theta$

$-dz' = a \sec^2\theta \, d\theta$

Integral then becomes $-\int \dfrac{a \sec^2\theta \, d\theta}{a^3 \sec^3\theta} = -\dfrac{1}{a^2} \int \cos\theta \, d\theta$

$$= \left[\frac{-\sin\theta}{a^2}\right] = -\frac{(z-z')}{a^2\left[a^2 + (z-z')^2\right]^{1/2}}$$

$$\underline{B} = \hat{\underline{z}} \frac{M a^2 \mu_0}{2 a^2} \left[\frac{z' - z}{\left[a^2 + (z-z')^2\right]^{1/2}}\right]_{-l/2}^{l/2}$$

$$= \hat{\underline{z}} \tfrac{1}{2} M \mu_0 \left[\frac{(\tfrac{1}{2}l - z)}{\left[a^2 + (z - \tfrac{1}{2}l)^2\right]^{1/2}} - \frac{(-\tfrac{1}{2}l - z)}{\left[a^2 + (z + \tfrac{1}{2}l)^2\right]^{1/2}}\right]$$

QED

(b) $\underline{H} = \dfrac{\underline{B}}{\mu_0} - \underline{M}$ with $\underline{B}$ given above.

(c) $l \ll a$ and $z \gg l$

$$\underline{B} = \hat{\underline{z}} \tfrac{1}{2} M \mu_0 \left[\frac{(\tfrac{1}{2}l - z)}{\left[a^2 + z^2 - zl + \tfrac{1}{4}l^2\right]^{1/2}} + \frac{(z + \tfrac{1}{2}l)}{\left[a^2 + z^2 + zl + \tfrac{1}{4}l^2\right]^{1/2}}\right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M \mu_0}{(a^2 + z^2)^{1/2}} \left[\frac{(\tfrac{1}{2}l - z)}{\left[1 - \frac{(zl - \tfrac{1}{4}l^2)}{(a^2 + z^2)}\right]^{1/2}} + \frac{(z + \tfrac{1}{2}l)}{\left[1 + \frac{(zl + \tfrac{1}{4}l^2)}{a^2 + z^2}\right]^{1/2}}\right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M \mu_0}{(a^2 + z^2)^{1/2}} \left[(\tfrac{1}{2}l - z)\left[1 + \frac{(zl - \tfrac{1}{4}l^2)}{2(a^2 + z^2)}\right] + (z + \tfrac{1}{2}l)\left[1 - \frac{(zl + \tfrac{1}{4}l^2)}{a^2 + z^2}\right]\right]$$

$$\underline{B} = \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0}{(a^2+z^2)^{1/2}} \left[\tfrac{1}{2}l - \cancel{z} + \cancel{z} + \tfrac{1}{2}l + \frac{(\tfrac{1}{2}l - z)(z - \tfrac{1}{4}l)l}{2(a^2+z^2)} - \frac{(z+\tfrac{1}{2}l)(z+\tfrac{1}{4}l)l}{2(a^2+z^2)} \right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0 l}{(a^2+z^2)^{1/2}} \left[1 + \left(\cancel{\tfrac{1}{2}lz} - \tfrac{1}{8}l^2 - z^2 + \cancel{\tfrac{1}{4}zl} - z^2 - \cancel{\tfrac{1}{4}lz} - \cancel{\tfrac{1}{2}lz} - \tfrac{1}{8}l^2 \right) / 2(a^2+z^2) \right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0 l}{(a^2+z^2)^{1/2}} \left[1 - \frac{(2z^2 + \tfrac{1}{4}l^2)}{2(a^2+z^2)} \right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0 l}{(a^2+z^2)^{3/2}} \left[\frac{2a^2 \cancel{+2z^2} \cancel{-2z^2} - \tfrac{1}{4}l^2}{2} \right]$$

$$\simeq \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0 l a^2}{(a^2+z^2)^{3/2}}$$

this being for $l \ll a$ and $z \gg l$

i.e. B_{out} with $z \gg l$

(d) $l \ll a$ and $|z| < \tfrac{1}{2}l$

$\Rightarrow$ $z \ll a$ also

$$\Rightarrow \underline{B} = \hat{\underline{z}} \tfrac{1}{2} M\mu_0 \left[\frac{(\tfrac{1}{2}l - z)}{a\left[1 + \frac{(z-\tfrac{1}{2}l)^2}{a^2}\right]^{1/2}} + \frac{(z+\tfrac{1}{2}l)}{a\left[1 + \frac{(z+\tfrac{1}{2}l)^2}{a^2}\right]^{1/2}} \right]$$

$$= \hat{\underline{z}} \frac{\tfrac{1}{2} M\mu_0}{a} \left[(\tfrac{1}{2}l - z)\left(1 - \frac{(z-\tfrac{1}{2}l)^2}{2a^2}\right) + (z+\tfrac{1}{2}l)\left(1 - \frac{(z+\tfrac{1}{2}l)^2}{2a^2}\right) \right]$$

$$= \hat{\underline{z}} \frac{M\mu_0}{2a} \left[\tfrac{1}{2}l - \cancel{z} + \cancel{z} + \tfrac{1}{2}l - \frac{(z+\tfrac{1}{2}l)^3}{2a^2} + \frac{(z-\tfrac{1}{2}l)^3}{2a^2} \right]$$

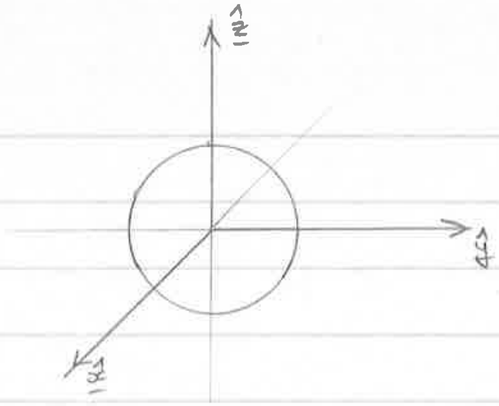
$$= \hat{\underline{z}} \frac{M\mu_0}{2a} \left[l + \frac{\left[\cancel{z^3} - \tfrac{1}{8}l^3 - 3z^2\tfrac{1}{2}l + \cancel{3z\tfrac{1}{4}l^2} - \cancel{z^3} - \tfrac{1}{8}l^3 - 3z^2\tfrac{1}{2}l - \cancel{3z\tfrac{1}{4}l^2}\right]}{2a^2} \right]$$

$$= \hat{\underline{z}} \frac{M\mu_0 l}{2a} \left[1 - \frac{(3z^2 + \tfrac{1}{4}l^2)}{2a^2} \right]$$

$$B_i \simeq \hat{\underline{z}} \frac{M\mu_0 l}{2a} \simeq 0$$

(e) $$H_i = \frac{B_i}{\mu_0} - M_i \simeq -M_i = -M\,\hat{z}$$

20-9)



$$\underline{M} = M(r)\,\hat{\underline{r}}$$

(a) $\underline{J}_M = \underline{\nabla} \wedge \underline{M}$

From above $M_r = M(r) \quad M_\theta = M_\phi = 0$
Using the form for $\underline{\nabla} \wedge \underline{M}$ in spherical co-ordinates since $M_r = M(r)$ (no θ, ϕ dependence)

2 $$\underline{\nabla} \wedge \underline{M} = 0$$
$$\Rightarrow \underline{J}_M = 0$$

(b) $\underline{K}_M = \underline{M} \wedge \hat{\underline{n}}$ where $\hat{\underline{n}} = \hat{\underline{r}}$

2 $= M(r)\,\hat{\underline{r}} \wedge \hat{\underline{r}}$

$= 0$

(c) $\underline{B}$ is given by $\underline{B} = \dfrac{\mu_0}{4\pi}\displaystyle\int_V \frac{\underline{J}_M \wedge \hat{\underline{R}}}{R^2}\,d\tau + \frac{\mu_0}{4\pi}\int_S \frac{\underline{K}_M \wedge \hat{\underline{R}}}{R^2}\,da$

2 when there are no free currents.
But since $\underline{J}_M$ and $\underline{K}_M$ are zero $\underline{B} = 0$ everywhere

2 (d) $\underline{H}_i = \dfrac{\underline{B}_i}{\mu_0} - \underline{M}_i = -\underline{M}_i = -\underline{M} = -M\,\hat{\underline{r}}$

1 (e) $\underline{B}_o = 0 \Rightarrow \underline{H}_o = -\underline{M}_o = 0$

Tangential b.c's are satisfied $\underline{H}_{ot} - \underline{H}_{it} = \underline{K}_f \wedge \hat{\underline{n}}$

$$\frac{\underline{B}_{ot}}{\mu_{out}} - \frac{\underline{B}_{it}}{\mu_{in}} = \underline{K}_f \wedge \hat{\underline{n}}$$

20-9-2

since $K_f = 0$

Normal b.c's $\quad B_{on} = B_{in}$ $\quad$ satisfied

$$\mu_{out} H_{on} = \mu_{in} H_{in}$$

$$0 = -\mu_{in} M(r)$$

$\Rightarrow M(a) = 0$

$\Rightarrow H_o(a) = 0$ $\quad$ since $H_i = -M\hat{r}$

20-17)

$$H\chi_{M\,mass} = \text{magnetic moment / unit mass}$$

$$H\chi_{M,molar} = \text{" " / mole}$$

$$H\chi_M = M = \text{magnetic moment / unit volume}$$

d = mass / unit volume

$$\Rightarrow \frac{M}{d} = \text{magnetic moment / mass} = \frac{H\chi_M}{d} = H\chi_{M,mass}$$

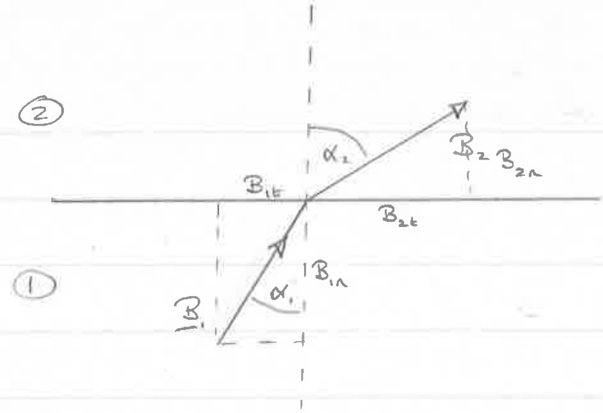
$$\Rightarrow \underline{\underline{\chi_M = d\chi_{M,mass}}}$$

A = mass / mole $\Rightarrow$ d/A = mole / volume

$$\Rightarrow \frac{M}{(d/A)} = \text{magnetic moment / mole} = \frac{H\chi_M A}{d} = H\chi_{M,molar}$$

$$\Rightarrow \chi_M = (d/A)\,\chi_{M,molar}$$

20-19)



(a) B.C.'s are $H_{2t} - H_{1t} = 0$

$$\frac{B_{2t}}{\mu_2} - \frac{B_{1t}}{\mu_1} = 0 \qquad \text{and} \quad B_{2n} = B_{1n}$$

with no free currents

$$\cot\alpha_1 = \frac{B_{1n}}{B_{1t}} \qquad \cot\alpha_2 = \frac{B_{2n}}{B_{2t}}$$

$$B_{1t}\cot\alpha_1 = B_{1n} \qquad B_{2t}\cot\alpha_2 = B_{2n}$$

3

$$B_{1t}\cot\alpha_1 = B_{2t}\cot\alpha_2$$

$$\mu_1\cot\alpha_1 = \mu_2\cot\alpha_2$$

$$\mu_0 K_1\cot\alpha_1 = \mu_0 K_2\cot\alpha_2$$

$$K_1\cot\alpha_1 = K_2\cot\alpha_2$$

(b) $|\chi_m| \ll 1 \Rightarrow \mu_1 \sim \mu_2 \sim \mu_0$ $\qquad \delta = (\alpha_2 - \alpha_1)$

$$\delta \simeq \sin\delta = \sin(\alpha_2 - \alpha_1) = \sin\alpha_2\cos\alpha_1 - \sin\alpha_1\cos\alpha_2$$

but

$$(1+\chi_1)\cos\alpha_1\sin\alpha_2 = (1+\chi_2)\cos\alpha_2\sin\alpha_1$$

from above

$$\Rightarrow \underbrace{\cos\alpha_1\sin\alpha_2 - \cos\alpha_2\sin\alpha_1}_{} = \chi_2\cos\alpha_2\sin\alpha_1 - \chi_1\cos\alpha_1\sin\alpha_2$$

3

$$\delta = \tfrac{1}{2}(\chi_2 - \chi_1)\sin 2\alpha_1$$

since $\alpha_1 \simeq \alpha_2$

When $\alpha_1 = 45°$ $\quad \chi_1 = 0 \quad \chi_2 = 2.2 \times 10^{-5}$

$$\delta = 0.5 \times 2.2 \times 10^{-5} \sin 90$$
$$= 1.1 \times 10^{-5} \text{ radians}$$
$$= 6.3 \times 10^{-4} \text{ degrees}$$

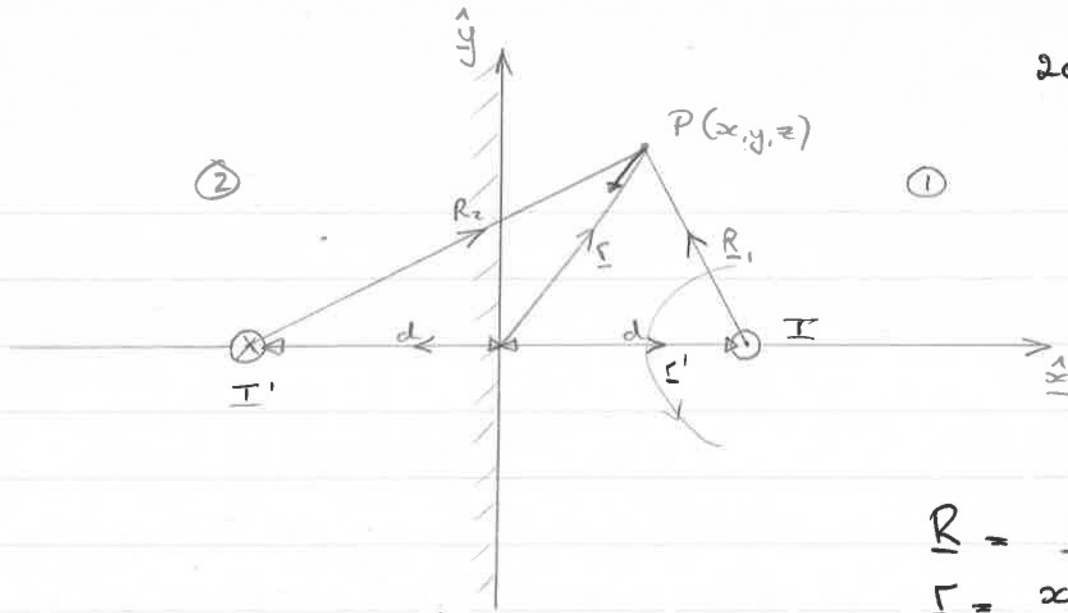
(c) (i) $|\chi_m|$ not small : $\alpha_1 = 45°$ $\quad \chi_1 = 0$
$\chi_2 = -0.95$

$$K_1 \cot \alpha_1 = K_2 \cot \alpha_2$$
$$(1 + \chi_1) \tan \alpha_2 = (1 + \chi_2) \tan \alpha_1$$
$$\tan \alpha_2 = (1 - 0.95) \tan 45$$
$$= 0.05$$
$$\alpha_2 = 2.86°$$
$$\delta = \alpha_2 - \alpha_1 = 2.86 - 45 = -42.1°$$

(ii) $\chi_2 = 500 \quad \chi_1 = 0 \quad \alpha_1 = 45°$

$$\tan \alpha_2 = (1 + \chi_2) \tan \alpha_1$$
$$\tan \alpha_1 = 501$$
$$\alpha_2 = 89.89°$$
$$\delta = \alpha_2 - \alpha_1 = 44.89°$$

20-23)



$$\underline{R} = \underline{r} - \underline{r}'$$
$$\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$$
$$\underline{r}' = d\hat{x}$$

(a) Ampère's Law $\oint \underline{H} \cdot d\underline{s} = I_{free,encl}$

$x > 0$

At P, due to I

$$H_1 2\pi\rho = I$$
$$H_1 = \frac{I}{2\pi\rho}\hat{\phi} = \frac{I}{2\pi R_1}\hat{\phi}$$
$$= \frac{I}{2\pi}\left[\frac{-\hat{x}\sin\phi + \hat{y}\sin\phi}{R_1}\right]$$

where $\sin\phi = y/R_1$ $\cos\phi = (x-d)/R_1$

$$\Rightarrow \underline{H}_1 = \frac{I}{2\pi}\left[\frac{-\hat{x}y + \hat{y}(x-d)}{R_1^2}\right]$$

Similarly due to I' $(\underline{H}_2)$

$$\underline{H}_2 = \frac{I'}{2\pi}\left[\frac{-\hat{x}y + \hat{y}(x+d)}{R_2^2}\right]$$

$$H_{TOT}^{x>0} = \underline{H}_1 + \underline{H}_2 = \frac{-\hat{x}}{2\pi}\left[\frac{Iy}{R_1^2} + \frac{I'y}{R_2^2}\right] + \frac{\hat{y}}{2\pi}\left[\frac{I(x-d)}{R_1^2} + \frac{I'(x+d)}{R_2^2}\right]$$

where $R_1^2 = (x-d)^2 + y^2 + z^2$

$R_2^2 = (x+d)^2 + y^2 + z^2$

For $x < 0$:

Image current is I'' positioned at $x = +d$

For any point $P(x, y, z)$ in region $x < 0$ we have

$$\underline{H}^{x<0} = \frac{I''}{2\pi} \frac{[-\hat{x} \sin\phi + \hat{y} \sin\phi]}{R_1}$$

$$= \frac{I''}{2\pi} \left[\frac{-\hat{x}\, y + \hat{y}(x-d)}{R_1^2} \right]$$

Normal B.C.'s are $\mu_2 H_{2n} = \mu_1 H_{1n}$ (normal components H_{1n}, H_{2n})

On this surface $\hat{x}$ components are normal.

$$\Rightarrow -\left(\frac{I y}{2\pi R_1^2} + \frac{I' y}{2\pi R_2^2} \right) \mu_1 = \frac{-I''}{2\pi} \frac{y}{R_1^2} \mu_2$$

where $R_1^2 = d^2 + z^2 + y^2$
and $R_2^2 = d^2 + z^2 + y^2$ } @ $x = 0$

$$\Rightarrow (I + I') \mu_1 = I'' \mu_2$$

But $\mu_1 = \mu_0$ and $\mu_2 = K_M \mu_0$

$$\Rightarrow \quad I + I' = I'' K_M \qquad \text{— (1)}$$

Tangential B.C.'s are $H_{2t} - H_{1t} = K_f$

$\hookrightarrow$ zero on surface

$$H_{2t} = H_{1t}$$

For $\hat{y}$ components @ $x = 0$

$$-\frac{I d}{2\pi R_1^2} + \frac{I' d}{2\pi R_2^2} = -\frac{I'' d}{2\pi R_1^2}$$

$$-I + I' = -I'' \qquad \text{— (2)}$$

Eliminating I' in (1) and (2) yields

$$2I = I''(K_m + 1)$$

$$I'' = \frac{2I}{(K_m+1)}$$

hence $I' = I - I'' = I - \frac{2I}{(K_m+1)}$

$$= I\frac{(K_m+1-2)}{(K_m+1)} = \frac{(K_m-1)I}{(K_m+1)}$$

(b) Force per unit length is found by evaluating force per unit length between I and I' distance $2d$ apart. Using the standard result for the force between two parallel currents

$$F = \frac{\mu_0 I I' l}{2\pi\, 2d}\,\hat{x}$$

[attractive I, I' like
repulsive $I I'$ opposite]

$$\Rightarrow F/l = \frac{\mu_0 I^2}{2\pi\, 2d}\left(\frac{K_m-1}{K_m+1}\right)$$

(c)

$$I' = I\frac{(K_m-1)}{(K_m+1)} = I\left(\frac{1+\chi_m-1}{1+\chi_m+1}\right) = \frac{\chi_m I}{\chi_m+2}$$

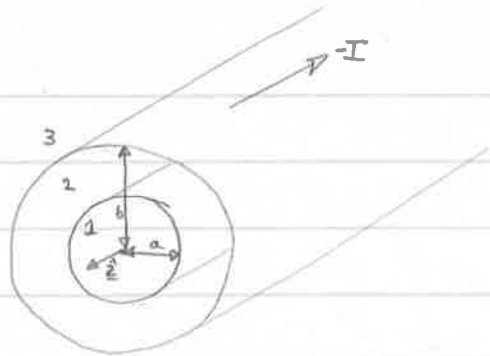
χ_m can be $>$ or $<$ 0

$\Rightarrow$ I' can be same or opposite to I

$\Rightarrow$ Force can be <u>attractive</u> or <u>repulsive</u>

[Disagrees with book]

20-25)



Medium 2 has

$K_m = K(\rho/a)$

(a) Apply ampère's Law around a cylinder radius ρ ($a<\rho<b$) co-axial with co-ax

$$\oint \underline{H} \cdot d\underline{s} = I_{enc}^{free}$$

2

$$H_\phi = \frac{I}{2\pi\rho}$$

(b) In this medium $\underline{B} = K_m \mu_0 \underline{H}$

2

$$\Rightarrow \quad B_\phi = \frac{K(\rho/a)\mu_0 I}{2\pi\rho} = \frac{\mu_0 K I}{2\pi a}$$

(c) Energy density $u_M = \frac{1}{2} H_\phi B_\phi$

$$= \frac{1}{2}\frac{I^2 \mu_0 K}{4\pi^2 \rho a}$$

$$U = \int u_M \, dv$$

6

$$= \frac{1}{2}\frac{I^2 \mu_0 K}{4\pi^2 a}\int \rho \frac{d\rho}{\rho} d\phi \, dz$$

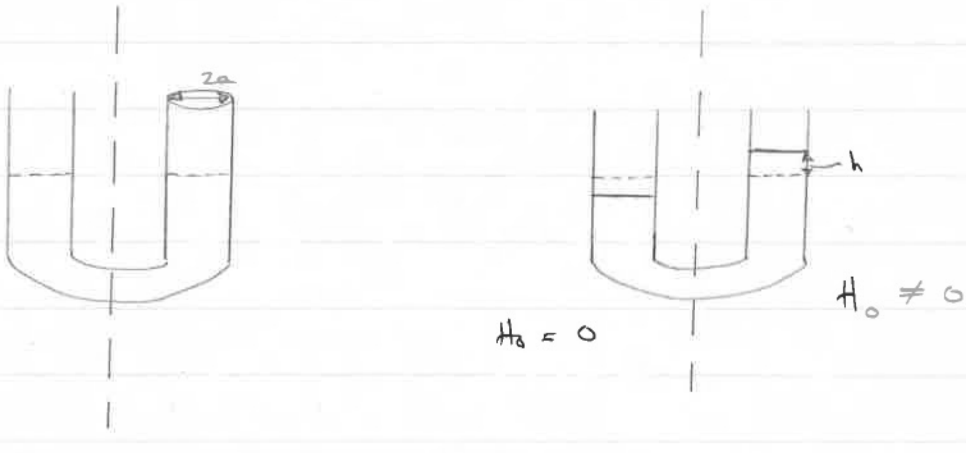
$$= \frac{1}{2}\frac{I^2 \mu_0 K \, 2\pi l(b-a)}{4\pi^2 a} = \frac{I^2 \mu_0 K l (b-a)}{2 \cdot 2\pi a}$$

Inductance $= \frac{1}{2} L I^2$

$\Rightarrow$ Contribution to L from this region is

$$\frac{\mu_0 K l (b-a)}{2\pi a}$$

20-27) [Similar to problem 10-37 in electrical case]



Energy conservation:

$$\Delta U\,(\text{gravitational}) \quad + \quad \Delta U\,(\text{magnetic}) = 0$$

(½ due to centre of gravity of raised liquid being at ½h. But other arm drops by ½h ⇒ $\Delta U_g = 0$?)

$$\tfrac{1}{2}\, g h d h \pi a^2 \quad + \quad \left(U_M^{\text{final}} - U_M^{\text{initial}}\right) = 0$$

$$g h^2 d \pi a^2 \quad + \quad \int_V \left(u_M^f - u_M^i\right) d\tau = 0$$

↳ volume occupied by height h of liquid

$$+ \tfrac{1}{2}\mu_o \int_V \left(K_m H_o^2 - H_o^2\right) d\tau = 0$$

$$+ \tfrac{1}{2}\mu_o H_o^2 \chi_m \pi a^2 h = 0$$

$$\Rightarrow \chi_m = -\frac{2gdh}{H_o^2 \mu_o}$$

N.B. The evaluation of the change in magnetic energy assumes that H_o is unaffected when the matter is introduced. Since we are told to ignore edge effects the problem is similar to the example on p 336 where the field $\underline{H}$ is created by a solenoidal current. [Of course we could have created H_o in this way]

Also note that for a paramagnetic material $K_M = \chi_M + 1$
with $\chi_M > 0 \Rightarrow K_M > 1$
(diamagnetic $\chi_M < 1 \Rightarrow K_M < 1$)

Thus paramagnetic $U^{final} > U^{initial} \Rightarrow$ energy needed
diamagnetic $U^{final} < U^{init} \Rightarrow$ energy released

$\Rightarrow$ Conclusion: For liquid to rise in $H_0 \neq 0$ region liquid must be diamagnetic
For liquid to rise in $H_0 = 0$ region liquid must be paramagnetic.

Disagrees with book