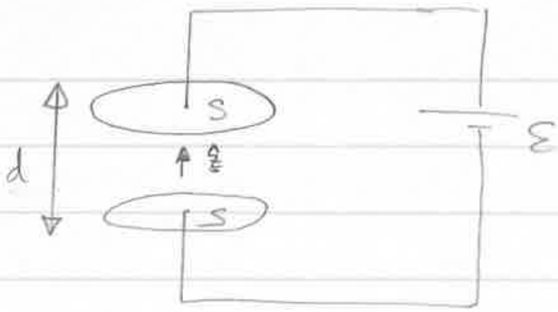
21-1)



$$d = d_0 + d_1 \sin\omega t$$

Constant voltage

(a) $$J_d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

Where $\int \frac{1}{2}\varepsilon_0 E^2 \, d\tau = \frac{1}{2} C \Delta\phi^2$ $\qquad C = \frac{A\varepsilon_0}{d}$

$$\frac{\cancel{\varepsilon_0} E^2 \cancel{S} d}{\cancel{2}} = \frac{\cancel{S}\cancel{\varepsilon_0}\,\mathcal{E}^2}{\cancel{2} d}$$

$$E = \frac{\mathcal{E}}{d}$$

$$\Rightarrow \quad \underline{J}_d = \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\mathcal{E}}{d}\right)\hat{\underline{z}}$$

$$\oint_c \underline{H}\cdot d\underline{s} = \int_s \underline{J}_d \cdot d\underline{a}$$

From symmetry $\underline{H}$ must be in $\hat{\phi}$ direction

$$H\, 2\pi\rho = \varepsilon_0 \mathcal{E} \frac{\partial}{\partial t}\left(\frac{1}{d}\right)\pi\rho^2$$

$$\underline{H} = \tfrac{1}{2}\varepsilon_0 \mathcal{E}\rho \, \partial/\partial t \left[d_0 + d_1\sin\omega t\right]^{-1} \hat{\phi}$$

$$= \tfrac{1}{2}\varepsilon_0 \mathcal{E}\rho \, \omega d_1 \cos\omega t \left[d_0 + d_1 \sin\omega t\right]^{-2} \hat{\phi}$$

(b) If battery is disconnected before oscillations begin we have a constant charge situation

$$\int \tfrac{1}{2}\varepsilon_0 E^2 \, d\tau = \frac{Q^2}{2C}$$

where $C_i = \dfrac{Q}{\Delta\phi_i} \Rightarrow Q = C_i \Delta\phi_i = \dfrac{S\varepsilon\Delta\phi_i}{d_0}$

therefore using $C = \dfrac{S\varepsilon_0}{d(t)}$

$$\cancel{\tfrac{1}{2}}\,\cancel{\varepsilon}\, E^2\, \cancel{S}\,\cancel{d} = \frac{\cancel{S^2}\,\cancel{\varepsilon_0^2}\,\Delta\phi_i^2\, \cancel{d(t)}}{\cancel{2}\, d_0^2\, \cancel{S}\,\cancel{\varepsilon_0}}$$

the $d(t)$ cancels leaving $E = \dfrac{\Delta\phi_i}{d_0} = \dfrac{\mathcal{E}}{d_0}$

E is constant $\Rightarrow$ $\underline{D} = \varepsilon_0 \underline{E}$ is also constant

and $\dfrac{\partial D}{\partial t} = 0 \Rightarrow \underline{J_d} = 0 \Rightarrow \underline{H}$ created by displacement current is also zero.

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or $D = q/A$

$\Rightarrow J_d = \dfrac{\partial D}{\partial t} = \dfrac{1}{A}\dfrac{dq}{dt}$

But q is constant when battery is disconnected

$\Rightarrow J_d = 0$

$\Rightarrow \underline{H} = 0$

21-3)

$$\underline{\nabla}\cdot\underline{D} = \rho_f \qquad \underline{\nabla}\cdot\underline{B} = 0$$

$$\underline{\nabla}\wedge\underline{E} = -\partial\underline{B}/\partial t \qquad \underline{\nabla}\wedge\underline{H} = \underline{J}_f + \partial\underline{D}/\partial t$$

and

$$\underline{D} = \varepsilon_0\underline{E} + \underline{P}$$

$$\underline{H} = \underline{B}/\mu_0 - \underline{M}$$

(a) $(\underline{E},\underline{H})$

using $\underline{D} = \underline{E}\varepsilon_0 + \underline{P}$ $\qquad \underline{\nabla}\cdot\underline{D} = \rho_f$

$$\varepsilon_0\underline{\nabla}\cdot\underline{E} = \rho_f - \underline{\nabla}\cdot\underline{P} \quad \text{— (1)}$$

$$\underline{B} = \mu_0\underline{H} + \mu_0\underline{M}$$

$$\Rightarrow \quad \underline{\nabla}\wedge\underline{E} = -\mu_0\,\partial\underline{H}/\partial t - \mu_0\,\partial\underline{M}/\partial t \quad \text{— (2)}$$

$$\underline{\nabla}\cdot\underline{B} = \mu_0\underline{\nabla}\cdot\underline{H} + \mu_0\underline{\nabla}\cdot\underline{M} = 0$$

$$\underline{\nabla}\cdot\underline{H} = -\underline{\nabla}\cdot\underline{M} \quad \text{— (3)}$$

$$\partial\underline{D}/\partial t = \varepsilon_0\,\partial\underline{E}/\partial t + \partial\underline{P}/\partial t$$

$$\Rightarrow \quad \underline{\nabla}\wedge\underline{H} = \underline{J}_f + \varepsilon_0\,\partial\underline{E}/\partial t + \partial\underline{P}/\partial t \quad \text{— (4)}$$

(b) $(\underline{D},\underline{B})$

$$\underline{\nabla}\cdot\underline{D} = \rho_f \quad \text{— (1)}$$

$$1/\varepsilon_0(\underline{\nabla}\wedge\underline{D}) - 1/\varepsilon_0(\underline{\nabla}\wedge\underline{P}) = -\partial\underline{B}/\partial t$$

$$\underline{\nabla}\wedge\underline{D} = -\varepsilon_0\,\partial\underline{B}/\partial t + \underline{\nabla}\wedge\underline{P} \quad \text{— (2)}$$

$$\underline{\nabla}\cdot\underline{B} = 0 \quad \text{— (3)}$$

$$(\underline{\nabla}\wedge\underline{B})/\mu_0 - \underline{\nabla}\wedge\underline{M} = \underline{J}_f + \partial\underline{D}/\partial t$$

$$\underline{\nabla}\wedge\underline{B} = \mu_0\underline{J}_f + \mu_0\,\partial\underline{D}/\partial t + \mu_0\underline{\nabla}\wedge\underline{M} \quad \text{— (4)}$$

(c) $(\underline{D},\underline{H})$

$$\underline{\nabla}\cdot\underline{D} = \rho_f \quad \text{— (1)}$$

$$1/\varepsilon_0(\underline{\nabla}\wedge\underline{D}) - 1/\varepsilon_0(\underline{\nabla}\wedge\underline{P}) = -\mu_0\left(\partial\underline{H}/\partial t + \partial\underline{M}/\partial t\right)$$

$$\underline{\nabla}\wedge\underline{D} = -\varepsilon_0\mu_0\left(\partial\underline{H}/\partial t + \partial\underline{M}/\partial t\right) + \underline{\nabla}\wedge\underline{P} \quad \text{— (2)}$$

$$\underline{\nabla}\cdot\underline{H} = -\underline{\nabla}\cdot\underline{M} \quad \text{— (3)}$$

$$\underline{\nabla}\wedge\underline{H} = \underline{J}_f + \partial\underline{D}/\partial t \quad \text{— (4)}$$

21-7) $\underline{\nabla} \cdot \underline{D} = \rho_f$ $\qquad$ $\underline{\nabla} \cdot \underline{B} = 0$

$\underline{\nabla} \wedge \underline{E} = -\partial \underline{B}/\partial t$ $\qquad$ $\underline{\nabla} \wedge \underline{H} = \underline{J}_f + \partial \underline{D}/\partial t$

$$\underline{D} = \varepsilon_0 \underline{E} + \underline{P}$$
$$\underline{H} = \underline{B}/\mu_0 - \underline{M}$$

In a linear isotropic <u>non-homogeneous</u> medium we can write

$$\underline{D} = \varepsilon(\underline{r})\underline{E} \quad \text{and} \quad \underline{B} = \mu(\underline{r})\underline{H}$$

i.e. ε and μ are functions of position

(a) $\underline{\nabla} \cdot \underline{D} = \rho_f$

$\underline{\nabla} \cdot (\varepsilon \underline{E}) = \rho_f$

3 $\underline{E} \cdot \underline{\nabla}\varepsilon + \varepsilon(\underline{\nabla} \cdot \underline{E}) = \rho_f$

$$\underline{\nabla} \cdot \underline{E} = (\rho_f - \underline{E} \cdot \underline{\nabla}\varepsilon)/\varepsilon$$

4 (b) $\underline{\nabla} \wedge \underline{E} = -\partial \underline{B}/\partial t$ $\qquad$ unchanged

4 (c) $\underline{\nabla} \cdot \underline{B} = 0$ $\qquad$ "

(d) $\underline{\nabla} \wedge \underline{H} = \underline{J}_f + \partial \underline{D}/\partial t$

$\underline{\nabla} \wedge (\underline{B}/\mu) = \underline{J}_f + \varepsilon\, \partial \underline{E}/\partial t$ $\qquad$ (ε, μ are not functions of t)

$\underline{\nabla}(1/\mu) \wedge \underline{B} + 1/\mu\,(\underline{\nabla} \wedge \underline{B}) = \underline{J}_f + \varepsilon\, \partial \underline{E}/\partial t$

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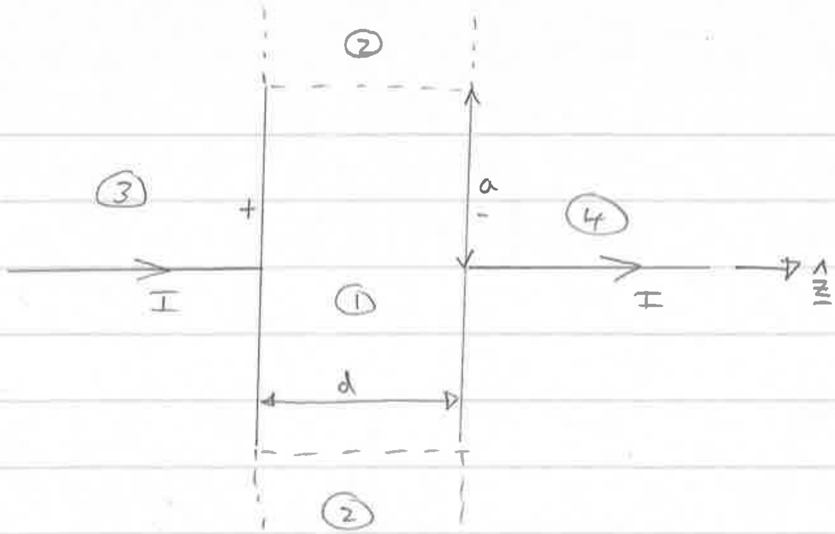
$$\underline{\nabla} \wedge \underline{B} = \mu \underline{J}_f + \mu\varepsilon\, \partial \underline{E}/\partial t - \mu\left(\nabla(1/\mu) \wedge \underline{B}\right)$$
$$= \mu \underline{J}_f + \mu\varepsilon \frac{\partial \underline{E}}{\partial t} + \mu\left(\frac{\underline{\nabla}\mu}{\mu^2} \wedge \underline{B}\right)$$
$$= \mu \underline{J}_f' + \mu\sigma \underline{E} + \mu\varepsilon \frac{\partial \underline{E}}{\partial t} + (\underline{\nabla}\mu \wedge \underline{B})/\mu$$

same as answer

21-9)



(a) $\underline{S} = \underline{E} \wedge \underline{H}$

In region ① $\quad \underline{H} = \dfrac{I\rho}{2\pi a^2}\,\hat{\phi}$ $\qquad$ (21-16)

In region ① $\quad \underline{E} = \dfrac{q}{\varepsilon_0 A}\,\hat{z}$

3 $\quad \underline{S} = \dfrac{qI\rho}{2\varepsilon_0 \pi a^2 \pi a^2}\,\hat{z} \wedge \hat{\phi}$

On bounding surface $\rho = a$

$$\Rightarrow \qquad \underline{S} = \frac{-qI}{2\varepsilon_0 \pi^2 a^3}\,\hat{\rho}$$

(no resistive heat loss)

(b) Total rate of energy flow into region ① is du/dt

$du/dt = \oint_{S'} \underline{S}\cdot\underline{da}$ $\quad$ over closed surface S' as shown above

[S' is a cylinder radius a height d]

4 Since $\underline{S}$ is along $\hat{\rho}$ contributions from ends are zero.

Thus $\quad \oint_{S'} \underline{S}\cdot\underline{da} = \displaystyle\int \frac{-qI}{2\varepsilon_0 \pi^2 a^3}\,\hat{\rho}\cdot\hat{\rho}\,\rho\, d\phi\, dz$ $\qquad (\rho = a)$

$$= \frac{-qI\,2\pi d}{2\varepsilon_0 \pi^2 a^2} = -\frac{qId}{\varepsilon_0 \pi a^2}$$

(– sign means energy is flowing into volume – see 21-58)

(c) Rate at which the energy of the capacitor is changing

3 $$u_{cap} = q^2/2c = \frac{q^2 d}{2 A \varepsilon_0} = \frac{q^2 d}{2\pi a^2 \varepsilon_0}$$

$$\frac{\partial u_{cap}}{\partial t} = \frac{d\, q\,(dq/dt)}{\pi \varepsilon_0 a^2} = \frac{d q I}{\pi \varepsilon_0 a^2}$$

= rate at which energy is flowing into the region from (b).

21-12)
$$\underline{E}' = c\left[\underline{E}\cos\alpha + (\mu\varepsilon)^{-1/2}\underline{B}\sin\alpha\right]$$
$$\underline{B}' = c\left[-(\mu\varepsilon)^{1/2}\underline{E}\sin\alpha + \underline{B}\cos\alpha\right]$$

$\underline{E}$, $\underline{B}$ are solutions to Maxwell's equations — prove that $\underline{E}'$, $\underline{B}'$ are also solutions.

(a)
$$\underline{\nabla}\cdot\underline{E}' = c\left[\underline{\nabla}\cdot\underline{E}\cos\alpha + (\mu\varepsilon)^{-1/2}\underline{\nabla}\cdot\underline{B}\sin\alpha\right] \quad -(1)$$

$$\underline{\nabla}\wedge\underline{E}' = c\left[\underline{\nabla}\wedge\underline{E}\cos\alpha + (\mu\varepsilon)^{-1/2}\underline{\nabla}\wedge\underline{B}\sin\alpha\right] \quad -(2)$$
$$\partial\underline{B}'/\partial t = c\left[-(\mu\varepsilon)^{1/2}\partial\underline{E}/\partial t\sin\alpha + \partial\underline{B}/\partial t\cos\alpha\right] \quad -(3)$$

$$\underline{\nabla}\cdot\underline{B}' = c\left[-(\mu\varepsilon)^{1/2}\underline{\nabla}\cdot\underline{E}\sin\alpha + \underline{\nabla}\cdot\underline{B}\cos\alpha\right] \quad -(4)$$

$$\underline{\nabla}\wedge\underline{B}' = c\left[-(\mu\varepsilon)^{1/2}\underline{\nabla}\wedge\underline{E}\sin\alpha + \underline{\nabla}\wedge\underline{B}\cos\alpha\right] \quad -(5)$$
6
$$\partial\underline{E}'/\partial t = c\left[\partial\underline{E}/\partial t\cos\alpha + (\mu\varepsilon)^{-1/2}\partial\underline{B}/\partial t\cos\alpha\right] \quad -(6)$$

Maxwells equations give us (when ρ_f and $\bar{J}_f$ are zero)
$$\underline{\nabla}\cdot\underline{E} = 0 \qquad \underline{\nabla}\cdot\underline{B} = 0$$

$$\underline{\nabla}\wedge\underline{E} = -\partial\underline{B}/\partial t \qquad \underline{\nabla}\wedge\underline{B} = \mu\varepsilon\,\partial\underline{E}/\partial t$$

Substituting in (1) $\quad \underline{\nabla}\cdot\underline{E}' = 0$

in (2) $\quad \underline{\nabla}\wedge\underline{E}' = c\left[-\partial\underline{B}/\partial t\cos\alpha + (\mu\varepsilon)^{-1/2}\mu\varepsilon\frac{\partial\underline{E}}{\partial t}\sin\alpha\right]$

$$\underline{\nabla} \wedge \underline{E}' = -\partial B'/\partial t \qquad \text{from (3)}$$

in (4) $\quad \underline{\nabla} \cdot \underline{B}' = 0$

in (5) $\quad \underline{\nabla} \wedge \underline{B}' = C\left[+(\mu\varepsilon)^{\frac{1}{2}} \partial B/\partial t \sin\alpha + \mu\varepsilon\, \partial E/\partial t \cos\alpha \right]$

$$= \mu\varepsilon \frac{\partial E'}{\partial t} \qquad \text{from (6)}$$

(b) $\quad \alpha = \pi/2$

$$\underline{E}' = C(\mu\varepsilon)^{-\frac{1}{2}} \underline{B}$$
$$\underline{B}' = -C(\mu\varepsilon)^{\frac{1}{2}} \underline{E}$$

$\Rightarrow$ $\underline{E}$ and $\underline{B}$ can be interchanged with addition of constants shown

v

(c) Simple sketch ?

(d) After (16-40) p257 $\qquad \underline{B} = \frac{\mu_0 \varepsilon_0 I}{\lambda} (\hat{\underline{z}} \wedge \underline{E})$

Where $\underline{B}$, $\underline{E}$, $\hat{\underline{z}}$ are mutually perpendicular

$$\underline{z} \wedge \underline{E}' = C(\mu\varepsilon)^{-\frac{1}{2}} \frac{\mu_0 \varepsilon_0 I}{\lambda} \hat{\underline{z}} \wedge (\hat{\underline{z}} \wedge \underline{E})$$

$$= C \frac{\mu_0 \varepsilon_0 I}{(\mu\varepsilon)^{\frac{1}{2}} \lambda} \left[\hat{\underline{z}} \underbrace{(\hat{\underline{z}} \cdot \underline{E})}_{\text{zero}} - \underline{E} \underbrace{(\hat{\underline{z}} \cdot \hat{\underline{z}})}_{\text{unity}} \right]$$

v

$$= -\frac{C \mu_0 \varepsilon_0 I \underline{E}}{\lambda (\mu\varepsilon)^{\frac{1}{2}}}$$

$$= \frac{C \mu_0 \varepsilon_0 I \underline{B}'}{\lambda (\mu\varepsilon)^{\frac{1}{2}} C (\mu\varepsilon)^{\frac{1}{2}}} = \frac{\mu_0 \varepsilon_0 I \underline{B}'}{\mu\varepsilon \lambda}$$

$$\Rightarrow \quad \underline{B}' = \frac{\mu \varepsilon \lambda}{\mu_0 \varepsilon_0 I} \left(\hat{\underline{z}} \wedge \underline{E}' \right)$$

which is the same form as our original unprimed relation.