24-7) $Q = \frac{\omega\varepsilon}{\sigma}$ $\quad v = \frac{1}{\sqrt{\mu\varepsilon}}\left[\frac{2}{\left[1+(1/Q^2)\right]^{1/2}+1}\right]^{1/2}$

$$= \frac{\omega}{\alpha}$$

$\delta = 1/\beta \qquad \tan\Omega = \beta/\alpha$

$$\frac{E}{Bc} = \frac{\omega}{|k|c}$$

where $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\left[1+\left(\frac{1}{Q^2}\right)^2\right]^{1/2}+1\right]}^{1/2}$

$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\left[1+\frac{1}{Q^2}\right]^{1/2}-1\right]}^{1/2}$

$|\underline{k}| = (\alpha^2+\beta^2)^{1/2}$

(a) Power: $v = 100$ Hz

$\omega = 2\pi v = 200\pi$

$Q = \frac{200\pi\varepsilon_0}{\sigma} = \frac{200\pi\, 8.85\times10^{-12}}{4} = \underline{\underline{1.39\times10^{-9}}}$

$Q \ll 1 \Rightarrow$ we can use good conductor approximation

$v = \left(\frac{2Q}{\mu\varepsilon}\right)^{1/2} = \left[\frac{2\times1.39\times10^{-9}}{4\pi\times10^{-7}\times8.85\times10^{-12}}\right]^{1/2} = 1.58\times10^{4}$ m/s

$\delta = \left(\frac{\cancel{2}}{\mu\sigma\cancel{2}\pi v}\right)^{1/2} = \left[\frac{1}{4\pi\times10^{-7}\times4\times\pi\times100}\right]^{1/2} = 25.2$ m

$E/cB = \frac{\omega}{\sqrt{2}.\alpha c} = \frac{\omega\sqrt{2}}{c\sqrt{2}.(\mu\sigma\omega)^{1/2}} = \left(\frac{\omega}{\sigma\mu}\right)^{1/2}\frac{1}{c}$

$= \left(\frac{2\pi\times100}{\cancel{4}\,8\times\cancel{4\pi}\times10^{-7}}\right)^{1/2}\frac{1}{3\times10^8} = 3.73\times10^{-5}$

$\tan\Omega = 1-Q$

$\Omega \simeq \pi/4$

(b) $\nu = 10^7$ Hz RADIO

$$Q = \frac{10^7\, 2\pi\, 8.85\times10^{-12}}{4} = 1.39\times10^{-4}$$

$Q \ll 1$

2

$$v = \left(\frac{2}{\mu\sigma}\right)^{1/2}(2\pi)^{1/2}\nu^{1/2} = 5\times10^6 \text{ m/s}$$

$$\delta = \left(\frac{2}{\mu\sigma 2\pi}\right)^{1/2}\frac{1}{\nu^{1/2}} = 7.96\times10^{-2}\text{ m}$$

$$E/cB = \frac{1}{c}\left(\frac{Q}{\mu\varepsilon}\right)^{1/2} = \frac{\cancel{c}}{\cancel{c}}(Q)^{1/2} = 1.18\times10^{-2}$$

$$\tan\Omega = 1 - Q \simeq 1 \Rightarrow \Omega = \pi/4$$

(c) Microwave: $\nu = 10^{10}$ Hz

$$Q = \frac{2\pi\, 10^{10}\times 8.85\times10^{-12}}{4} = 1.39\times10^{-1}$$

$Q \sim 1 \Rightarrow$ we must use exact expressions

2

$$\alpha = \frac{2\pi\nu}{c}\frac{1}{\sqrt{2}}\left[\left(1+\frac{1}{Q^2}\right)^{1/2}+1\right]^{1/2} = 4.26\times10^2$$

$$\beta = \frac{2\pi\nu}{c}\frac{1}{\sqrt{2}}\left[\left(1+\left(\frac{1}{Q^2}\right)\right)^{1/2}-1\right]^{1/2} = 3.71\times10^2$$

$$v = \frac{\omega}{\alpha} = \frac{2\pi\nu}{\alpha} = 1.47\times10^8 \text{ m/s}$$

$$\delta = \frac{1}{\beta} = 2.69\times10^{-3}\text{ m}$$

$$E/cB = \frac{\cancel{c}}{\cancel{c}}\left(1+\frac{1}{Q^2}\right)^{-1/4} = 0.371$$

$\tan\Omega = \beta/\alpha \Rightarrow \Omega = 0.716$ rad.

(d) Light $\nu = 10^{15}$ Hz

$$Q = \frac{10^{15} \times 2 \times \pi \times 8.85 \times 10^{-12}}{4} = 1.39 \times 10^{4}$$

$Q \gg 1$

2

$$v = c\left(1 - \frac{1}{8Q^2}\right) = c \simeq 3 \times 10^{8} \text{ m/s}$$

$$\delta = \frac{2Q}{\omega\sqrt{\mu\varepsilon}} = \frac{2Qc}{2\pi\nu} = 1.33 \times 10^{-3} \text{ m}$$

$$E/cB = \frac{1}{c\sqrt{\mu\varepsilon}}\left(1 + \frac{1}{4Q^2}\right) = \left(1 + \frac{1}{4Q^2}\right) \simeq 1$$

$$\tan\Omega = \frac{1}{2Q} \Rightarrow \Omega = 3.6 \times 10^{-5} \text{ rad}$$

(e) δ for light $= 1.33$ mm

Ridiculous because light penetrates sea-water by many metres.

(f) Because we have used the static values of ε, μ, σ — in particular σ.

24-11) $\langle u_m \rangle / \langle u_e \rangle$

(a) $\langle u_e \rangle = \frac{1}{2}\varepsilon \langle E^2 \rangle$; $\langle u_m \rangle = \frac{1}{2}\mu \langle H^2 \rangle$

$$= \frac{1}{2\mu} \langle B^2 \rangle$$

$$\underline{E} = \underline{E}_{0a} e^{-\beta z} e^{i(\alpha z - \omega t + \theta)} \quad \text{--- } (24\text{-}61)$$

$$= \underline{E}_{0a} e^{-\beta z}\left[\cos(\alpha z+\theta) + i\sin(\alpha z+\theta)\right]\left[\cos\omega t - i\sin\omega t\right]$$

$$= \underline{E}_{0a} e^{-\beta z}\left[\cos(\alpha z+\theta)\cos\omega t + \sin(\alpha z+\theta)\sin\omega t + i\left(\cos\omega t \sin(\alpha z+\theta) - \sin\omega t\cos(\alpha z+\theta)\right)\right]$$

$$E_{actual} = \underline{E}_{0a} e^{-\beta z}\left[\cos(\alpha z+\theta)\cos\omega t + \sin(\alpha z+\theta)\sin\omega t\right]$$

$$\langle E^2 \rangle = E_{0a}^2 e^{-2\beta z} \cdot \frac{1}{2}$$

$$\underline{B} = \frac{|k|}{\omega}\hat{\underline{z}} \wedge \underline{E}_{0a} e^{-\beta z} e^{i(\alpha z - \omega t + \theta + \Omega)} \quad (24\text{-}61)$$

$$\underline{B}_{actual} = \frac{|k|}{\omega}\hat{\underline{z}} \wedge E_{0a} e^{-\beta z}\left[\cos(\alpha z+\theta+\Omega)\cos\omega t + \sin(\alpha z+\theta+\Omega)\sin\omega t\right]$$

$$\langle B^2 \rangle = \frac{k^2}{\omega^2} E_{0a}^2 e^{-2\beta z} \frac{1}{2}$$

$$\Rightarrow \langle u_m \rangle / \langle u_e \rangle = \frac{k^2}{2\omega^2\mu} \frac{E_{0a}^2 e^{-2\beta z} \frac{1}{2}}{E_{0a}^2 e^{-2\beta z} \frac{1}{2}\frac{1}{2}\varepsilon} = \frac{k^2}{\omega^2\mu\varepsilon}$$

where $|k| = \omega\sqrt{\mu\varepsilon}\left(1 + \frac{1}{Q^2}\right)^{1/4}$

$$\Rightarrow \frac{\langle u_m \rangle}{\langle u_e \rangle} = \frac{\omega^2\mu\varepsilon\left(1 + 1/Q^2\right)^{1/2}}{\omega^2\mu\varepsilon} = \left(1 + \frac{1}{Q^2}\right)^{1/2}$$

(b) "Insulator" ($Q \gg 1$) $\left(1 + \frac{1}{Q^2}\right)^{1/2} \simeq 1 + \frac{1}{2Q^2} + \cdots$

$$= 1 + \frac{1}{2Q^2}$$

(c) Conductor $Q \ll 1$

$$\left(1 + \frac{1}{Q^2}\right)^{1/2} = \frac{1}{Q}\left[1 + Q^2\right]^{1/2}$$

$$= \frac{1}{Q}\left[1 + \frac{Q^2}{2} + \cdots\right]$$

$$= \frac{1}{Q}\left[1 + \frac{Q^2}{2}\right]$$

Non conductor

$$\underline{B} = \frac{\hat{z} \wedge \underline{E}}{c} \qquad c = \omega / k$$

Conductor

$$\underline{B} = \frac{(\hat{z} \wedge E)}{v} \cancel{k} = \frac{(\hat{z} \wedge \underline{E})}{\omega} k$$

but $k \Rightarrow |k| e^{i\Omega}$

$$\underline{B} = \frac{|k|}{\omega} e^{i\Omega} (\hat{z} \wedge \underline{E})$$

Non Conductor $\dfrac{|E|}{|B|} = c$

~~Non~~ conductor

$$\frac{|E|}{|B|} = \frac{\omega}{|k| e^{i\Omega}} = \frac{\omega}{(\alpha^2+\beta^2)^{1/2}} e^{-i\Omega}$$

Thus

$$\frac{\langle u_e \rangle}{\langle u_B \rangle} = \frac{\mu \varepsilon E^2}{B^2} = \mu\varepsilon \frac{\omega^2}{\underbrace{(\alpha^2+\beta^2)}_{Q \text{ dependence.}}}$$

$\Rightarrow \langle u_e \rangle \neq \langle u_B \rangle$ in conductors

For conductors $Q \ll 1$ $\alpha = \beta = \left(\frac{\mu\sigma\omega}{2}\right)^{1/2} \Rightarrow \langle u_e \rangle \ll \langle u_B \rangle$

non $Q \gg 1$ $\beta = 0$ $\alpha = \omega\sqrt{\mu\varepsilon}$ $\Rightarrow \langle u_e \rangle = \langle u_B \rangle$

$$\frac{\langle u_e \rangle}{\langle u_B \rangle} = \frac{\mu\varepsilon\omega^2 2}{2\mu\sigma\omega} = \frac{\omega\varepsilon}{\sigma} = Q \quad (\ll 1)$$

$$\frac{\langle u_E \rangle}{\langle u_B \rangle} \ll 1 \quad \text{or} \quad \langle u_B \rangle \gg \langle u_e \rangle$$