25-11) (a) $$\frac{E_{ot}}{E_{oi}} = \frac{2}{1+(\mu_1/\mu_2)(k_2/k_1)} = \frac{2}{1+(\mu_1/\mu_2)(c/n_1\omega)(\alpha_2+i\beta_2)}$$

(25-82)

Thus $$\frac{E_{ot}}{E_{oi}} = \frac{2}{\left[1+(\mu_1/\mu_2)(c/n_1\omega)\alpha_2 + i(\mu_1/\mu_2)(c/n_1\omega)\beta_2\right]}$$
$$= \frac{2\left[1+(\mu_1/\mu_2)(c/n_1\omega)\alpha_2 - i(\mu_1/\mu_2)(c/n_1\omega)\beta_2\right]}{\left[1+(\mu_1/\mu_2)(c/n_1\omega)\alpha_2\right]^2+\left[(\mu_1/\mu_2)(c/n_1\omega)\beta_2\right]^2}$$

From Chapter 24 p387 we find that for a good conductor

$$\left.\begin{aligned} \alpha_2 &= (\tfrac{1}{2}\mu_2\sigma_2\omega)^{1/2}(1+\tfrac{1}{2}Q) \\ \beta_2 &= 1/\delta_2 = (\tfrac{1}{2}\mu_2\sigma_2\omega)^{1/2}(1-\tfrac{1}{2}Q) \\ \lambda_2 &= 2\pi\left(\frac{2}{\mu_2\sigma_2\omega}\right)^{1/2}(1-\tfrac{1}{2}Q) \end{aligned}\right\} **$$

with $Q = \omega\varepsilon/\sigma$

and $\lambda_1 = \dfrac{2\pi c}{\omega n_1} \Rightarrow \dfrac{c}{\omega n_1} = \dfrac{\lambda_1}{2\pi}$

Therefore $\dfrac{E_{ot}}{E_{oi}}$ becomes

$$\left(\frac{E_{ot}}{E_{oi}}\right) = \frac{2\left[1+(\mu_1/\mu_2)(\lambda_1/2\pi)\alpha_2 - i(\mu_1/\mu_2)(\lambda_1/2\pi)\beta_2\right]}{\left[1+(\mu_1/\mu_2)(\lambda_1/2\pi)\alpha_2\right]^2+\left[(\mu_1/\mu_2)(\lambda_1/2\pi)\beta_2\right]^2}$$

For a very good conductor we can approximate the above ** relations even further

$$\alpha_2 = \beta_2 = 1/\delta_2$$

and so

$$\left(\frac{E_{ot}}{E_{oi}}\right) = \frac{2\left[1+(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2) - i(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2)\right]}{\left[1+(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2)\right]^2+\left[(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2)\right]^2}$$

The expression $\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\lambda_1}{2\pi}\right)\left(\frac{1}{\delta_2}\right)$

is critical to the evaluation of (E_{0t}/E_{0i}) and can be re-written as

$$\frac{\mu_1}{\mu_2}\cdot\frac{\cancel{2\pi} c}{\omega n_1 \cancel{2\pi}}\cdot\frac{(\mu_2\sigma\omega)^{1/2}}{\sqrt{2}} = \frac{\mu_1}{\mu_2}\frac{c}{n_1}\left(\frac{\sigma\mu_2}{2\omega}\right)^{1/2}$$

$$= \frac{\mu_1}{\mu_2}\frac{c}{n_1}\left(\frac{\sigma}{2\omega\varepsilon_2}\right)^{1/2}\frac{1}{v_2} = \frac{\mu_1}{\mu_2}\frac{c}{n_1}\left(\frac{1}{2Q_2}\right)^{1/2}\frac{1}{\sqrt{2\ Q_2}}\ \frac{1}{v_{2\,noncond}}$$

(by 24-77)

$$= \frac{\mu_1}{\mu_2}\frac{\cancel{c}}{n_1}\cdot\frac{1}{Q_2}\frac{1}{2}\frac{n_{2\,noncond}}{\cancel{c}}$$

$$= \frac{\mu_1}{\mu_2}\frac{1}{Q_2\,2\,n_1}n_{2\,noncond}$$

$\Rightarrow$ p423 Example

Assuming now that $\mu_1 \sim \mu_2$ and $n_1 \lessapprox n_{2\,noncond}$ then if $Q_2 \ll 1$ (true for a conductor) then this term will be $\gg 1$

$\Rightarrow$ In the expression for E_{0t}/E_{0i} we may ignore the "1"

$$\Rightarrow \quad \frac{E_{0t}}{E_{0i}} = \frac{2\,(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2)[1-i]}{2\left[(\mu_1/\mu_2)(\lambda_1/2\pi)(1/\delta_2)\right]^2}$$

$$= \frac{\mu_2}{\mu_1}\frac{2\pi}{\lambda_1}\delta_2(1-i)$$

as expected

(b) $\frac{E_{0t}}{E_{0i}} = \frac{2\pi\mu_2\delta_2}{\lambda_1\mu_1}\sqrt{2}\cdot e^{-i\pi/4}$

Therefore the phase difference between E_{0t} and E_{0i} is $\pi/4 = 45^\circ$

(c)

We may write $E_i = E_{i0} \cos(kz - \omega t + \phi_i)$
and $E_t = E_{t0} \cos(kz - \omega t + \phi_t)$
for waves travelling in z direction.

Using $P = kz - \omega t + \phi_i$
and $\phi_i - \phi_t = \Delta$ = phase difference

$E_i = E_{i0} \cos P$

$E_t = E_{t0} \cos(P - \Delta)$

If $\Delta < 0$ E_t lags E_i
$\Delta > 0$ E_t leads E_i } see Fig 24-12.

In our case Δ is > 0 $(+\pi/4)$

$\Rightarrow E_t$ leads E_i by 45°.

25-12) Good conductor

From (25-83) we have

$$R \simeq 1 - 2\frac{\mu_2}{\mu_1}\frac{n_1}{n_2}$$

But $\lambda_1 = \dfrac{2\pi c}{\omega n_1} \Rightarrow n_1 = \dfrac{2\pi c}{\lambda_1 \omega}$

and $n_2 = \dfrac{c\alpha_2}{\omega}$

But $\alpha_2 \simeq \beta_2 \simeq 1/\delta_2$ for a good conductor

Therefore $n_2 = \dfrac{c}{\omega \delta_2}$

and
$$R \simeq 1 - 2\frac{\mu_2}{\mu_1}\frac{2\pi c\,\omega\delta_2}{\lambda_1 \omega c}$$
$$\simeq 1 - 4\pi\frac{\mu_2}{\mu_1}\frac{\delta_2}{\lambda_1}$$

25-13)

(a) (25-71) $T = \left| \frac{\langle \underline{S}_t \rangle \cdot \hat{\underline{n}}}{\langle \underline{S}_i \rangle \cdot \hat{\underline{n}}} \right| = \frac{Z_1 \cos\theta_t}{Z_2 \cos\theta_i} \left| \frac{E_t}{E_i} \right|^2$

For normal incidence $\theta_i = 0° = \theta_t$

From (24-104)

$$\langle \underline{S} \rangle = \tfrac{1}{2} Re\left(\underline{E}_c \wedge H_c^* \right)$$

2 where E_c and H_c are the complex forms of $\underline{E}$ and $\underline{H}$

For plane waves $\underline{E}_c$ and H_c are @ 90° and when $\theta_i = 0°$ they are both perpendicular to $\hat{\underline{n}}$

$$\Rightarrow \quad T = \frac{\langle \underline{S}_t \rangle \cdot \hat{\underline{n}}}{\langle \underline{S}_i \rangle \cdot \hat{\underline{n}}} = \frac{Re\left(E_{ct} H_{ct}^*\right)}{Re\left(E_{ci} H_{ci}^*\right)}$$

as expected

Using (25-82) $\frac{E_{ot}}{E_i} = \frac{2}{1 + (\mu_1/\mu_2)(1/k_1)\, 1/\delta_2\, (1+i)}$

where we have substituted $k_1 = \frac{n_1 \omega}{c} \left(= \frac{2\pi}{\lambda_1} \right)$

and the good conductor approximation $\alpha_2 \simeq \beta_2 = 1/\delta_2$

then $\frac{E_{ot}}{E_i} = \frac{2}{1 + A(1+i)}$ where $A = \frac{\mu_1}{\mu_2} \frac{\lambda_1}{2\pi} \frac{1}{\delta_2}$

$$= \frac{2(1 + A - Ai)}{(1+A)^2 + A^2}$$

6 A is exactly the expression we found equal to $\frac{\mu_1}{\mu_2} \frac{n_2}{n_1} \frac{1}{Q_2} \frac{1}{2}$

in problem 25-11. For a good conductor $Q \ll 1$ and assuming $n_2 \gtrsim n_1$ and $\mu_1 \sim \mu_2$ $A \gg 1$

$$\Rightarrow \quad \frac{E_{ot}}{E_i} \simeq \frac{2A(1-i)}{2A^2} = \frac{1-i}{A}$$

Thus
$$\left|\frac{E_{0t}}{E_{0i}}\right|^2 = \frac{(1-i)(1+i)}{A^2} = \frac{2}{A^2}$$
$$= \frac{2\,\mu_2^2\, 4\pi^2\, \delta_2^2}{\mu_1^2\, \lambda_1^2}$$
$$\Rightarrow \quad T = \frac{Z_1}{Z_2}\, 2\left(\frac{\mu_2 2\pi \delta_2}{\mu_1 \lambda_1}\right)^2$$

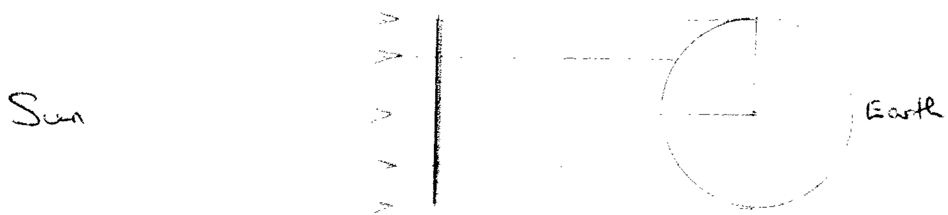
now
$$\frac{Z_1}{Z_2} = \left(\frac{\mu_1 \varepsilon_2}{\varepsilon_1 \mu_2}\right)^{1/2} = \left(\frac{\mu_1}{\mu_2}\right)^{1/2}\frac{v_1\, \mu_1^{1/2}}{v_2\, \mu_2^{1/2}} = \frac{\mu_1}{\mu_2}\frac{c}{n_1}\underbrace{\left(\frac{\mu_2 \sigma}{2\omega}\right)^{1/2}}_{\substack{24\text{-}77 \\ \text{good conductor}}}$$

$$\frac{Z_1}{Z_2} = \frac{\mu_1}{\mu_2}\frac{c}{n_1}\left(\frac{\mu_2 \sigma \omega}{2\omega^2}\right)^{1/2}$$
$$= \frac{\mu_1}{\mu_2}\frac{c}{n_1}\frac{1}{\omega \delta_2} \qquad (\text{using } 24\text{-}78)$$
$$= \frac{\mu_1}{\mu_2 \delta_2}\frac{\lambda_1}{2\pi} \qquad (25\text{-}78)$$
$$\Rightarrow \quad T = \frac{\mu_1}{\mu_2}\frac{\lambda_1}{\delta_2 2\pi}\, 2\left(\frac{\mu_2}{\mu_1}\frac{2\pi}{\lambda_1}\delta_2\right)^2 = 2\frac{\mu_2}{\mu_1}\frac{2\pi}{\lambda_1}\delta_2$$
$$= 4\pi\frac{\mu_2}{\mu_1}\frac{\delta_2}{\lambda_1}$$
as expected

(b) $(25\text{-}75)$ $\quad R + T = 1$
$$\Rightarrow \quad R = 1 - T$$
$$= 1 - 4\pi\frac{\mu_2}{\mu_1}\frac{\delta_2}{\lambda_1}$$

which is exactly the result of the previous exercise 25-12.

25-16) Assuming that the earth reflects no radiation ($R = 0$) then
$$P(\Theta_i) = \cos^2\Theta_i \langle u_i \rangle$$



Since the earth is so far from the sun we can safely assume that the radiation from the sun forms a parallel beam as shown above.

In addition I am going to assume that all the radiation is at normal incidence – i.e. the earth is a flat disk of radius R_e. This, of course, is not the case. In the real situation the radiation is incident at varying Θ_i, but this approximation will only introduce factors of order unity. [N.B. the actual case will look something like

$$dP = \cos^2\Theta \langle u_i \rangle \, d\Omega$$
$$P = \int \cos^2\Theta \sin\Theta \, d\Theta \, d\phi \, \langle u_i \rangle$$
$$= 2\pi \, \tfrac{1}{3} \langle u_i \rangle$$
$$= \frac{2\pi}{3} \langle u_i \rangle$$
]

$\Rightarrow$ Force due to radiation $F_R = P \pi R_e^2$
$$= \langle u_i \rangle \pi R_e^2$$

but $\langle u_i \rangle = \langle S \rangle / c$ for em radiation in a vacuum.

Now we assume the value of $\langle S \rangle = 1340$ watts/m^2 which allows us to evaluate F_R.

25 - 16 - 2

F due to gravity $F_G = \dfrac{G M_e M_s}{R_{SE}^2}$

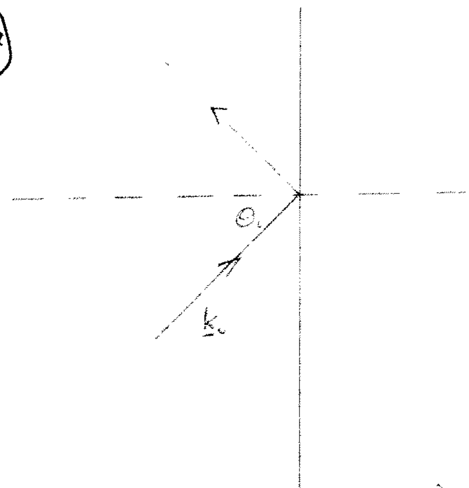
$$\Rightarrow \quad \frac{F_R}{F_G} = \frac{\langle S \rangle \pi R_e^2 R_{se}^2}{c\, G M_e M_s}$$

Using
$$R_e = 6.37 \times 10^6 \text{ m}$$
$$R_{se} = 1.49 \times 10^{11} \text{ m}$$
$$c = 3 \times 10^8 \text{ m/s}$$
$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2$$
$$M_s = 2 \times 10^{30} \text{ kg}$$
$$M_e = 5.98 \times 10^{24} \text{ kg}$$

$$\frac{F_R}{F_G} = \frac{1340 \times \pi \times (637 \times 10^6)^2 \times (1.49 \times 10^{11})^2}{3 \times 10^8 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 2 \times 10^{30}} = \frac{3.792 \times 10^{39}}{2.393 \times 10^{40} \times 10^{+3}}$$

$$= 1.58 \times 10^{-14}$$

25-17)



$$P(\theta_i) = 2\cos^2\theta_i \langle u_i \rangle$$

If surface is isotropically radiated we may write

$$dP = 2\cos^2\theta \langle u_i \rangle \, d\Omega$$

$$P_{TOT} = 2\langle u_i \rangle \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta \, d\phi$$

using $u = \cos\theta$

$$= 2\langle u_i \rangle \, 2\pi \int_0^1 u^2 \, du$$

$$= \frac{4\pi}{3} \langle u_i \rangle$$

But $dU_{TOT} = 2\langle u_i \rangle \, d\Omega$ (including incident and reflected radiation)

$$U_{TOT} = 4\pi \langle u_i \rangle$$

$$\Rightarrow \quad P_{TOT} = \tfrac{1}{3} U_{TOT}$$