Physics 542 – Test 2 Important Formulas and Topics

Potentials (Ch.22)

$\vec{B} = \vec{\nabla} \wedge \vec{A}$ $\quad \vec{E} = -\vec{\nabla}\phi - \partial\vec{A}/\partial t$

Lorentz condition: $\vec{\nabla}\cdot\vec{A} + \mu\varepsilon \, \partial\phi/\partial t + \mu\sigma\phi = 0$ $\quad$ d'Alembertian: [box]$^2 = \nabla^2 - \mu\varepsilon\, \partial^2/\partial t^2$

Gauge Transformations: $\vec{A}' \to \vec{A} + \nabla\chi$ $\quad \phi' \to \phi - \partial\chi/\partial t$

Waves (Ch.24-25)

Wave equation from Maxwell's equations. 1 D wave equation: $\partial^2\psi/\partial z^2 = 1/c^2 \, \partial^2\psi/\partial t^2$

Plane wave solution: $\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$ $\quad$ similarly for **B.**

$k = 2\pi/\lambda$ $\quad \upsilon = \nu\lambda = \omega/k$ $\quad n = c/\upsilon$ $\quad Z = \left(\mu/\varepsilon\right)^{1/2}$ $\quad c = \left(\mu_0\varepsilon_0\right)^{-1/2}$

Proof that waves are transverse with **E** and **B** orthogonal.

Waves in conducting media: $\vec{E} = \vec{E}_0 e^{-\beta z}\, e^{i(k\alpha-\omega t)}$

Complex refractive index. Role of $Q = \omega\varepsilon/\sigma$ $\quad$ Skin depth, $\delta = 1/\beta$

Polarization. Proof of laws of reflection ($\theta_i = \theta_r$) and refraction (Snell's law: $n_1 \sin\theta_i = n_2 \sin\theta_r$)

Brewster's Law/Angle: $\tan\theta_B = n_2/n_1$ from Fresnel equations.

TIR and evanescent waves properties.

Reflection coefficient - $R = \left|\langle\vec{S}_r\rangle\cdot\hat{n}\right| / \left|\langle\vec{S}_i\rangle\cdot\hat{n}\right|$ $\quad$ Transmission coefficient- $T = \left|\langle\vec{S}_t\rangle\cdot\hat{n}\right| / \left|\langle\vec{S}_i\rangle\cdot\hat{n}\right|$

Radiation pressure (vacuum), $P(\theta_i) = \dfrac{(1+R)}{c}\cos^2\theta_i \left|\langle\vec{S}_i\rangle\right|$

Momentum density (vacuum), $\langle g\rangle = \langle S\rangle / c^2$

Energy density expressions: $\langle u_e\rangle = \left\langle \tfrac{1}{2}\varepsilon E^2\right\rangle = \tfrac{1}{4}\varepsilon E_c \cdot E_c^*$ $\quad \langle u_m\rangle = \left\langle \tfrac{1}{2}\mu H^2\right\rangle = \tfrac{1}{4}\mu H_c \cdot H_c^*$

$$\langle u\rangle = \langle u_e\rangle + \langle u_m\rangle = \tfrac{1}{2}\varepsilon\left|E_0\right|^2 = \tfrac{1}{2}\mu\left|H_0\right|^2 = \left|\langle\vec{S}\rangle\right|/v$$