# VECTOR FORMULAS

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} \tag{1-29}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \tag{1-30}$$

$$\nabla\times\nabla u = 0 \tag{1-48}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \tag{1-49}$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}) \tag{1-106}$$

$$\frac{d}{d\sigma}(u\mathbf{A}) = \frac{du}{d\sigma}\mathbf{A} + u\frac{d\mathbf{A}}{d\sigma} \tag{1-107}$$

$$\frac{d}{d\sigma}(\mathbf{A}\cdot\mathbf{B}) = \frac{d\mathbf{A}}{d\sigma}\cdot\mathbf{B} + \mathbf{A}\ \frac{d\mathbf{B}}{d\sigma} \tag{1-108}$$

$$\frac{d}{d\sigma}(\mathbf{A}\times\mathbf{B}) = \frac{d\mathbf{A}}{d\sigma}\times\mathbf{B} + \mathbf{A}\times\frac{d\mathbf{B}}{d\sigma} \tag{1-109}$$

$$\nabla(u+v) = \nabla u + \nabla v \tag{1-110}$$

$$\nabla(uv) = u\nabla v + v\nabla u \tag{1-111}$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{B}\times(\nabla\times\mathbf{A}) + \mathbf{A}\times(\nabla\times\mathbf{B}) + (\mathbf{B}\cdot\nabla)\mathbf{A} + (\mathbf{A}\cdot\nabla)\mathbf{B} \tag{1-112}$$

$$\nabla(\mathbf{C}\cdot\mathbf{r}) = \mathbf{C} \qquad \text{where } \mathbf{C} = \text{const.} \tag{1-113}$$

$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B} \tag{1-114}$$

$$\nabla\cdot(u\mathbf{A}) = \mathbf{A}\cdot(\nabla u) + u(\nabla\cdot\mathbf{A}) \tag{1-115}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{1-116}$$

$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B} \tag{1-117}$$

$$\nabla\times(u\mathbf{A}) = (\nabla u)\times\mathbf{A} + u(\nabla\times\mathbf{A}) \tag{1-118}$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\nabla\cdot\mathbf{B})\mathbf{A} - (\nabla\cdot\mathbf{A})\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} \tag{1-119}$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{1-120}$$

*where*

$$(\mathbf{A}\cdot\nabla)\mathbf{B} = \hat{\mathbf{x}}\left(A_x\frac{\partial B_x}{\partial x} + A_y\frac{\partial B_x}{\partial y} + A_z\frac{\partial B_x}{\partial z}\right) + \hat{\mathbf{y}}\left(A_x\frac{\partial B_y}{\partial x} + A_y\frac{\partial B_y}{\partial y} + A_z\frac{\partial B_y}{\partial z}\right) + \hat{\mathbf{z}}\left(A_x\frac{\partial B_z}{\partial x} + A_y\frac{\partial B_z}{\partial y} + A_z\frac{\partial B_z}{\partial z}\right) \tag{1-121}$$

# VECTOR OPERATIONS

## RECTANGULAR COORDINATES

$$\nabla u = \hat{\mathbf{x}}\frac{\partial u}{\partial x} + \hat{\mathbf{y}}\frac{\partial u}{\partial y} + \hat{\mathbf{z}}\frac{\partial u}{\partial z} \tag{1-37}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{1-42}$$

$$\nabla\times\mathbf{A} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \tag{1-43}$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \tag{1-46}$$

## CYLINDRICAL COORDINATES

$$\nabla u = \hat{\boldsymbol{\rho}}\frac{\partial u}{\partial \rho} + \hat{\boldsymbol{\varphi}}\frac{1}{\rho}\frac{\partial u}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial u}{\partial z} \tag{1-85}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \tag{1-87}$$

$$\nabla\times\mathbf{A} = \hat{\boldsymbol{\rho}}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \hat{\boldsymbol{\varphi}}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \hat{\mathbf{z}}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\varphi) - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right] \tag{1-88}$$

$$\nabla^2 u = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial\varphi^2} + \frac{\partial^2 u}{\partial z^2} \tag{1-89}$$

## SPHERICAL COORDINATES

$$\nabla u = \hat{\mathbf{r}}\frac{\partial u}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial u}{\partial\theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial u}{\partial\varphi} \tag{1-101}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} \tag{1-103}$$

$$\nabla\times\mathbf{A} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right] + \frac{\hat{\boldsymbol{\varphi}}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] \tag{1-104}$$

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2} \tag{1-105}$$