Physics 541 – Important Formulas and Topics

Basic Mathematics

Divergence Theorem: $\oint_S \boldsymbol{A}\cdot \boldsymbol{da} = \int_V (\nabla\cdot\mathbf{A})\,d\tau$

Stokes' Theorem: $\oint_C \mathsf{A}\cdot \mathsf{ds} = \int_S (\nabla\wedge A)\cdot da$

Gradient definition: $du = ds\cdot\nabla u$

Cylindrical co-ordinates: $r = \rho\hat{\rho} + z\hat{z}$; $\partial\hat{\rho}/\partial\varphi = \hat{\varphi}$; $\partial\hat{\varphi}/\partial\varphi = -\hat{\rho}$; $d\tau = d\rho\rho d\varphi dz$

$\hat{\rho} = \hat{x}\cos\varphi + \hat{y}\sin\varphi$; $\hat{\varphi} = -\hat{x}\sin\varphi - \hat{y}\cos\varphi$; $\hat{z} = \hat{z}$
$\hat{x} = \hat{\rho}\cos\varphi - \hat{\varphi}\sin\varphi$; $\hat{y} = \hat{\rho}\sin\varphi + \hat{\varphi}\cos\varphi$; $\hat{z} = \hat{z}$

Spherical co-ordinates: $r = r\hat{r}$ $\quad \partial\hat{r}/\partial\theta = \hat{\theta}$; $\partial\hat{r}/\partial\varphi = \hat{\varphi}\sin\theta$ $\quad d\tau = dr\,rd\theta\,r\sin\theta d\varphi$

$\partial\hat{\theta}/\partial\theta = -\hat{r}$; $\partial\hat{\theta}/\partial\varphi = \hat{\varphi}\cos\theta$

$\partial\hat{\varphi}/\partial\theta = 0$; $\partial\hat{\varphi}/\partial\varphi = -\hat{r}\sin\theta + \hat{\theta}\cos\theta$

$\hat{r} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$; $\hat{\theta} = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta$; $\hat{\varphi} = -\hat{x}\sin\varphi + \hat{y}\cos\varphi$
$\hat{x} = \hat{r}\sin\theta\cos\varphi + \hat{\theta}\cos\theta\cos\varphi - \hat{\varphi}\sin\varphi$; $\hat{y} = \hat{r}\sin\theta\sin\varphi + \hat{\theta}\cos\theta\sin\varphi + \hat{\varphi}\cos\varphi$; $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$

Electrostatics

Coulomb: $\vec{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qq'}{R^2}\hat{\vec{R}} \;=\; q\dfrac{1}{4\pi\varepsilon_0}\int_{V'}\hat{\vec{R}}\dfrac{\rho(\vec{r}\,')}{R^2}d\tau'$; $\vec{F} = q\vec{E}$

Gauss: $\oint_S \vec{E}\cdot da = \sum q_{in}/\varepsilon_0 = \int_V \rho/\varepsilon_0\, d\tau$; $\nabla\cdot E = \rho/\varepsilon_0$

Scalar Potential: $\phi(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0}\int_V \dfrac{\rho(\vec{r}\,')}{R}d\tau'$; $\vec{E} = -\vec{\nabla}\phi$

$$\Delta\phi = -\int_1^2 \vec{E}\cdot ds \quad ; \quad \vec{\nabla}\wedge\vec{E} = 0 \quad ; \quad \oint_C \vec{E}\cdot ds = 0$$

In electrostatics, **E** = 0 in conductors

Electrostatic energy: $U = \frac{1}{2}\int\limits_{all\,space} \rho(r)\phi(r)\,d\tau = \frac{1}{2}\int\limits_{all\,space} \varepsilon_0 E^2\,d\tau = \frac{1}{2}\int\limits_{all\,space} u_e\,d\tau$

Capacitors: $C = \frac{Q}{\Delta\phi}$ Parallel plate capacitor: $C = \frac{A\varepsilon_0}{d}$

Energy stored in capacitors: $U = \frac{1}{2}C(\Delta\phi)^2 = Q^2/2C = \frac{1}{2}Q(\Delta\phi)$

Monopole moment: $Q_{mon} = \int_V \rho\,d\tau$ Dipole moment: $\vec{p} = \int_{V'} \rho(r')r'\,d\tau' = \sum_i q_i\vec{r}_i$

Dipole scalar potential: $\phi_{dip} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$

Dipole in external E field: $U = -\vec{p}\cdot\vec{E}$ $\vec{\tau} = \vec{p}\wedge\vec{E}$

Boundary Conditions: $E_{1t} = E_{2t}$ $E_{2n} - E_{1n} = \sigma/\varepsilon_0$ $\phi_1 = \phi_2$

Polarization: $dp = \vec{P}(\vec{r})d\tau$ $\rho_b = -\nabla\cdot\vec{P}$ $\sigma = \vec{P}\cdot\hat{n}$

ϕ due to **P** is sum of ϕ given by ρ_b and σ_b

Displacement Field: $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$: b.c's

$D_{2n} - D_{1n} = \sigma_f$ $D_{2t} - D_{1t} = P_{2t} - P_{1t}$

Gauss Law in dielectric media: $\nabla\cdot\vec{D} = \rho_f$

In l.i.h media $\vec{P} = \chi_e\varepsilon_0\vec{E}$ *where* $\chi_e = \kappa_e - 1$ $\varepsilon = \varepsilon_0\kappa_e$

Effect of media on E, C, $\Delta\phi$ etc.

Energy in dielectrics (l.i.h): $u_e = \frac{1}{2}D\cdot E = \frac{1}{2}\varepsilon E^2 = \frac{D^2}{2\varepsilon}$

Current: $I = dq/dt = \int_S J\cdot da = \int_C (K\cdot\hat{t})\,ds$

Current density: $\vec{J} = \rho\vec{v}$ $\vec{K} = \sigma\vec{v}$ $I = \lambda|\vec{v}|$

Current elements: $\vec{J}d\tau \equiv \vec{K}da \equiv \boldsymbol{Ids} \equiv q\vec{v}$ Conduction current: $\vec{J} = \sigma\vec{E}$

Equation of continuity: $\nabla\cdot J = -\partial\rho/\partial t$

Resistance & Power: $\Delta\phi = IR$ $R = \frac{\rho\ell}{A}$ $Power = I^2R$

Ampere's Law: $F_{CC'} = \frac{\mu_0}{4\pi}\oint_C\oint_{C'} \frac{Ids \wedge (I'ds' \wedge \hat{R})}{R^2} = \frac{-\mu_0 II'}{4\pi}\oint_C\oint_{C'} \frac{(ds \cdot ds')\hat{R}}{R^2}$

Definition of the Ampere

Biot-Savart Law: $\vec{B} = \frac{\mu_0}{4\pi}\oint_{C'} \frac{I'ds' \wedge \hat{R}}{R^2}$ so that $\vec{F} = \oint_C I\, ds \wedge \vec{B}$

(similarly for surface and volume currents)

Ampere's Law Integral form: $\oint_C B \cdot ds = \mu_0 I_{encl}$ Differential form: $\nabla \wedge \vec{B} = \mu_0 \vec{J}$

No magnetic monopoles: $\nabla \cdot \vec{B} = 0$

B b.c's: $B_{2t} - B_{1t} = \mu_0 (K \wedge \hat{n})$ $B_{2n} - B_{1n} = 0$

Vector Potential: $\vec{B} = \nabla \wedge \vec{A}$ $\vec{A} = \frac{\mu_0}{4\pi}\oint_{C'} \frac{I'ds'}{R}$ similarly for $Jd\tau$, **K**da and q**v**

$\nabla \cdot A = 0$ $\nabla^2 A = -\mu_0 J$ $A\ continuous$

$$\Phi_B = \int_S B \cdot da = \oint_C A \cdot ds$$

Gauge transformation $A \to A + \nabla\chi$ $where\ \nabla^2\chi = 0$

Faraday's Law: $\varepsilon_{ind} = -\frac{d\Phi_B}{dt} = \oint_C E_{ind} \cdot ds$ $\nabla \wedge E = -\frac{\partial B}{\partial t}$

Mutual Induction:

$$M_{jk} = \frac{\mu_0}{4\pi}\oint_{C_j}\oint_{C_k} \frac{ds_j \cdot ds_k}{R_{jk}} \qquad \varepsilon_j^{ind} = -\sum_k M_{jk}\frac{dI_k}{dt} \qquad \Phi_j = \sum_k M_{jk} I_k$$

Self Induction: $L = \frac{\mu_0}{4\pi}\oint_j\oint_j \frac{ds_j \cdot ds_j}{R_{jj}}$ $\varepsilon_{ind} = -L\frac{dI}{dt}$ $\Phi_B = LI$

Magnetic Energy: $U_B = \tfrac{1}{2}\sum_j \oint_{C_j} A(r_j) \cdot ds_j I_j = \tfrac{1}{2}\int_V (J \cdot A)\, d\tau = \tfrac{1}{2} LI^2 = \int_V \frac{B^2}{2\mu_0} d\tau$

Magnetic energy density: $u_B = \frac{B^2}{2\mu_0}$

Magnetic forces on circuits: $F_B = -\nabla U_B = -II'\nabla M$

Magnetic Multipoles: $\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}}{R} d\tau = \vec{A}_{mon}(\vec{r}) + \vec{A}_{Dip}(\vec{r}) + \vec{A}_{Quad}(\vec{r}) + \ldots$

where, $\vec{A}_{Mon} = \frac{\mu_0}{4\pi r}\int_V \vec{J}\, d\tau' = 0$ $\quad A_{Dip} = \frac{\mu_0}{4\pi r^2}\int_V \vec{J}\,(\hat{r}\cdot r')\,d\tau' = \frac{\mu_0}{4\pi r^2}(m \wedge \hat{r})$

and magnetic dipole moment (filamentary currents): $\vec{m} = \frac{1}{2}\oint_C (\vec{r}' \wedge ds')I' \equiv I\vec{S}$

Multipole field from $B = \nabla \wedge A$

PE of mag dipole in external B field: $U_B = -\vec{m}\cdot\vec{B}_{ext}$

Torque on magnetic dipole in external B field: $\vec{\tau} = \vec{m} \wedge \vec{B}_{ext}$

Net force on magnetic dipole: $F = -\nabla U_B$