University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Fall 2014

Paper E – Contemporary Physics

Time allowed – 40 minutes each section

Instructions and Information:

- ***Attempt any 2 of the 6 questions***
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Nuclear & Particle Physics

(a) List three basic properties an elementary particle must have to be classified as a baryon. (15)

(b) The delta baryon resonances (Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-}) are the four members of a mulitplet all of which decay to a nucleon and a pion. What are the values of the isospin and strangeness of the Δ resonance ? Justify your conclusions. (15)

(c) The width of the Δ is 115 MeV. Estimate its lifetime. (26)

(d) Indicate, with reasons, which interaction (Strong, Weak or Electromagnetic) you would expect to be responsible for the following reactions:

(i) $\pi^{+} + n \Rightarrow p + n + \overline{n} + \pi^{0}$ (11)

(ii) $\Xi^{-} \Rightarrow \Lambda^{0} + \pi^{-}$ (11)

(iii) $\Sigma^{0} \Rightarrow \Lambda^{0} + \gamma$ (11)

(iv) $e^{+} + e^{-} \Rightarrow \mu^{+} + \mu^{-}$ (11)

Some elementary particle properties

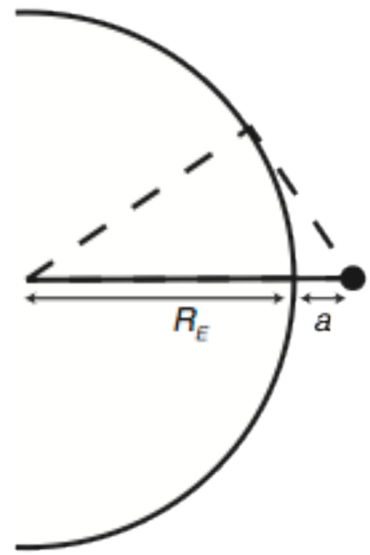
	p	**n**	**π^-**	**π^0**	**Σ^0**	**Ξ^-**	**e^-**	**μ^-**	**γ**	**Λ^0**
(Q/e) Charge	+1	0	-1	0	0	-1	-1	-1	0	0
Baryon #	+1	+1	0	0	+1	+1	-	-	-	+1
Lepton #	-	-	-	-	-	-	+1	+1	-	-
Strangeness	0	0	0	0	-1	-2	-	-	-	-1
Mass MeV/c^2	938	940	140	135	1192	1321	0.51	106	0	1115

Antiparticle properties can be found by multiplying by -1

Atmospheric Physics

(a) Calculate the flux density (or irradiance) over a hemisphere for a flat surface emitting with an isotropic intensity (i.e. $I(\theta, \phi)=I$). (15)

(b) If the surface in (a) is a blackbody, what is the intensity as a function of blackbody temperature? (10)



(c) Assume that the earth is a spherical blackbody (radius R_E) with an effective temperature of T_E. A spherical satellite with radius r_{sat} , also an ideal blackbody, is located an altitude, a, above the surface of the Earth. Calculate the flux of radiation received by the satellite from a differential solid angle of the Earth.
Hint: Use plane parallel assumption for the incoming radiation from a differential solid angle. (15)

(d) Integrate over the appropriate solid angle to calculate the total incoming radiation for the satellite from the Earth. (15)

(e) Expand the flux relation in (d) for small $R_E/(R_E + a)$ and express the result using the luminosity of the earth. For small $R_E/(R_E + \mathrm{a})$, what is the physical interpretation? (25)

(f) Using the exact and approximate results from (d) and (e) for the incoming flux, calculate the radiative equilibrium temperature of the satellite for a low earth orbit (a = 1700 km) and a geostationary orbit (a = 32000km). T_E = 255K, R_E = 6370 (20)

Optics

The reflectivity of materials can be adjusted with coating layers.

(a) Anti-reflection coatings reducing the reflectivity of a substrate are common in optical elements. Determine the reflectivity of $n_s = 1.52$ glass coated with a $\lambda/4$ thick layer of MnF ($n_l = 1.35$). Absorption and multiple reflections may be neglected. (40)

Hint: the amplitude of the electrical field reflected at 90° incidence at an interface between two materials with indices of refraction $n_{a,b}$ is $E_{refl} = E_{inc}\frac{n_a - n_b}{n_a + n_b}$.

(b) What is the reflectivity in the limit when $n_l \to 1$? (10)

(c) What is the reflectivity in the limit when $n_l \to n_s$? (10)

(d) At what value of n_l is the reflectivity a minimum? (10)

(e) In the previous example the reflectivity of the surface was decreased by adding a film. Show how the reflectivity of a material can be *increased* by coating with a different type of $\lambda/4$ layer. What should the index of refraction n_l of the film be relative to that of the substrate n_s: larger or smaller? (20)

(f) A substrate coated with one type of layer on one side and another type on the other side can be made to reflect, and therefore to transmit, light differently depending on the direction of light propagation. Give a few examples. (10)

Atomic & Molecular Physics

Optical Tweezers

(a) Consider an atom of mass m that emits its energy E as photon of (rest) frequency f_0 such that $E = hf_0$. The photon is then measured in a laboratory to have a frequency f. At what velocity and in what direction did the atom recoil? (25)

(b) If the atom is at rest in the laboratory before it emits the photon, what frequency f is observed in the laboratory for the emitted light? (25)

(c) In the limit of low velocity, use conservation of momentum to calculate what frequency is observed if the atom is moving away from the measuring instrument at speed v. How does this compare to the "Doppler shift"? (25)

(d) Instead of an atom, consider a small transparent sphere in a "Gaussian" laser beam. That is, a beam of photons in which there are more photons at the center than than at edge with a Gaussian profile. Explain why the sphere will feel a force directing it toward the axis of the laser beam. (Hint: The sphere acts like a lens, focusing light that passes through it.) (25)

Astrophysics

Temperature of a Planet

The solar constant, the irradiance of the Earth with light from the Sun, has been measured by Earth satellites to be about 1.36 kilowatts per square meter above the atmosphere. The Stefan-Boltzmann constant σ is 5.67 x 10^{-8} W m^{-2} K^{-4}.

Consider the Earth and its satellite the Moon, both at the same average distance from the Sun. For the following you do not need to know either the radius of the Earth or the Moon.

(a) What would the temperature of the illuminated surface of the Moon be if it were a perfectly absorbing blackbody? Assume that the Moon rotates so slowly the illuminated surface comes to thermal equilibrium before it goes into darkness again. (25)

(b) The Moon's "albedo" averaged over the solar spectrum is about 0.11, that is, it reflects about 11% of the light that strikes it. What would be the equilibrium temperature of the illuminated lunar surface taking into account that not all of the light is absorbed. (25)

(c) The Earth rotates rapidly by comparison, and its atmosphere redistributes the incident solar energy around the globe. The average albedo of the Earth is higher than the Moon at about 0.3 . What would you expect the average temperature of the Earth's surface to be and how does this compare to the freezing point of water at atmospheric pressure? Explain your assumptions about how effectively the warm Earth can radiate into space. (25)

(d) Asphalt has an albedo of about 0.04. Use this and your results for part c to comment on why urban areas of Earth would have "heat islands". (25)

Condensed Matter Physics

This problem concerns the tight-binding energies of a crystal with the hexagonal Bravais lattice symmetry. The α-orbital tight-binding energy $E_\alpha\left(\vec{k}\right)$ is given by $E_\alpha(\vec{k}) = \varepsilon_\alpha - J_\alpha^0 - \sum_n J_\alpha^1(\vec{R}_n)e^{-i\vec{k}\cdot\vec{R}_n}$, where α represents the atomic orbital, ε_α is the atomic orbital energy (e.g., ε_s, the s-orbital energy, ε_p, the p-orbital energy, etc.), J_α^0, the on-site potential energy, $J_\alpha^1(\vec{R}_s)$, the off-site potential energy, $\vec{k}$, the reciprocal vector, and $\vec{R}_n$, the nearest neighbor lattice vectors. For a given orbital α, ε_α and J_α^0 are constants, and the tight-binding energies $E_\alpha\left(\vec{k}\right)$ will depend on the symmetry of the crystal via the third term in the equation.

(*To answer this problem it is not critical that you understand exactly what is meant by the tight-binding energy of a crystal. What you need to do in this problem is to know how to use it to study the crystal with the hexagonal lattice symmetry. The basic knowledge that you need is the hexagonal symmetry, the primitive lattice (or basis) vectors, the nearest neighbor lattice vectors, and the reciprocal lattice vectors.)*

(a) Fig. 1 (below) shows the hexagonal Bravais lattice. The solid circles represent points (or atoms) forming the Bravais lattice. Write down the primitive lattice vectors ($\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$) (i.e., the black-bold arrows shown in Fig. 1) in terms of the lattice constant a, c, and the unit vectors ($\hat{i}$, $\hat{j}$, and $\hat{k}$) in Cartesian coordinates. Namely, you need to find the x-, y-, and z-components for each vector, and express it as $\vec{a}_i = a_{ix}\hat{i} + a_{iy}\hat{j} + a_{iz}\hat{k})$, i=1, 2, 3. (20)

(b) Find the primitive reciprocal lattice vectors ($\vec{b}_1$, $\vec{b}_2$, and $\vec{b}_3$) for the hexagonal lattice. The primitive reciprocal lattice vectors ($\vec{b}_1$, $\vec{b}_2$, and $\vec{b}_3$) are defined as $\vec{b}_1 = \frac{2\pi\left(\vec{a}_2\times\vec{a}_3\right)}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)}$, $\vec{b}_2 = \frac{2\pi\left(\vec{a}_3\times\vec{a}_1\right)}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)}$, and $\vec{b}_3 = \frac{2\pi\left(\vec{a}_1\times\vec{a}_2\right)}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)}$. Namely, you need to substitute $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ obtained from (a) into these formulas for $\vec{b}_1$, $\vec{b}_2$, and $\vec{b}_3$. (20)

(c) Find the coordinates of the 8 nearest neighbor lattice vectors $\vec{R}_n = (R_{nx}, R_{ny}, R_{nz})$, n=1, 2, …, 8, for the hexagonal lattices with respect to the origin (Fig. 1). Namely, you need to find the components of the vectors starting from the origin and ending on the nearest neighbor points in the hexagonal lattice. (20)

(d) Using the formula $E_\alpha\left(\vec{k}\right)$ given in the first paragraph of the problem and the results from (c) to express the s-orbital (i.e., α=s) tight binding energy $E_s\left(\vec{k}\right)$ of the hexagonal as a function of $\vec{k}$. (Hint: $J_s^1(\vec{R}_n) = J_s^1$ is independent of $\vec{R}_n$ when the orbital has a spherical symmetry, like s-orbital, and ε_α and J_α^0 are constants) (20)

(e) Using the results from (d) to find the s-orbital tight binding energy $E_s\left(\vec{k}\right)$ for the hexagonal in terms of ε_α, J_α^0 , and J_s^1 at Γ point (i.e., $\vec{k}=(0,0,0)$) and K point (i.e., $\vec{k}=\frac{\pi}{a}(\frac{2}{3},\frac{1}{3},0)$, respectively. (20)

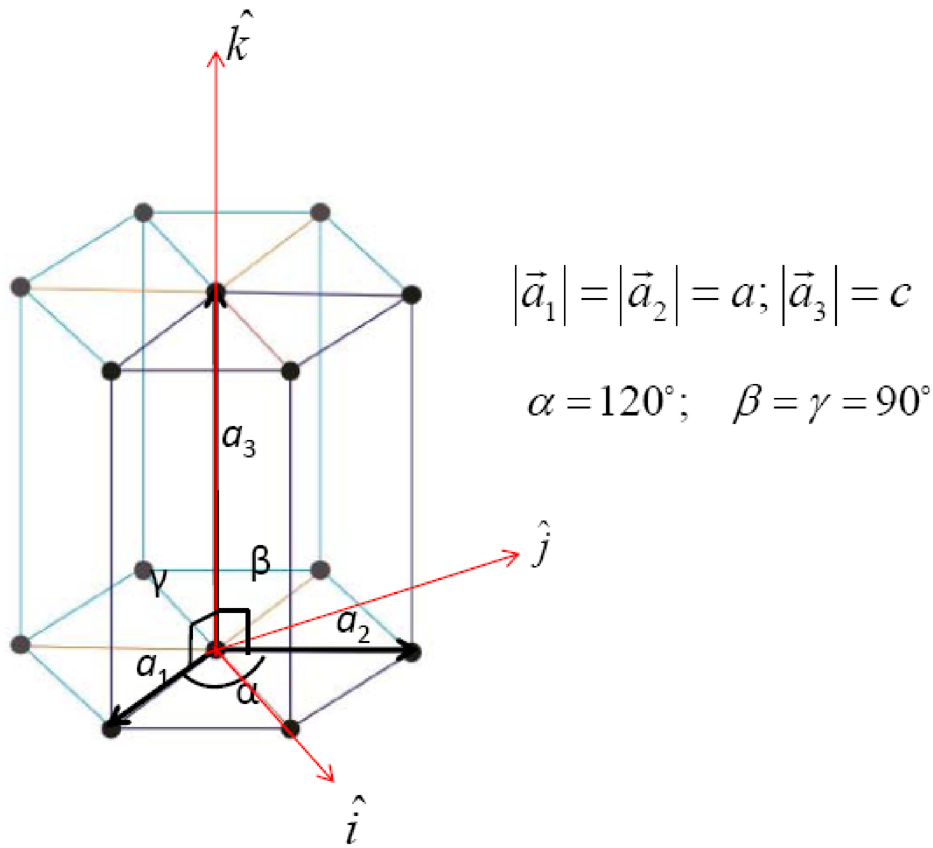


Fig. 1 The hexagoanl Bravais lattice. The solid circles represent the points (or atoms) forming the lattice. The black-bold arrows are the primitive lattice vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$. The thin black arrows denote the unit vectors of the Cartesian coordinates. The origin is located at the center of the hexagon.