University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Fall 2014

Paper D – Quantum Mechanics

Time allowed – 90 minutes

Instructions and Information:

- Answer both questions
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Quantum Mechanics Basic Level

A particle, which is confined to move in one-dimension between 0 and L, is described by the wave function

$$\psi(x) = Ax(L - x)$$

(a) Use the normalization condition to determine the constant A. (8)

(b) Derive an expression for the average value of the position of the particle. (9)

(c) Write down expressions for the operators representing the momentum and the kinetic energy of the particle. (8)

(d) Derive an expression for the average value of the kinetic energy of the particle. (10)

Quantum Mechanics Intermediate Level

A particle of mass m is in the state $|\Psi(x,t)\rangle = Ae^{-a\left[\left(mx^2/\hbar\right)+it\right]}$, where A and a are positive real constants.

(a) Evaluate the constant A. (10)

(b) For what potential energy function V(x) does $|\Psi(x,t)\rangle$ satisfy the Schrödinger equation? (10)

(c) Calculate the expectation values of $\hat{x}$, $\hat{x}^2$, $\hat{p}_x$ and $\hat{p}_x^2$ (20)

(d) Find the standard deviations σ_x and σ_{p_x}. Is their product $\sigma_x\sigma_{p_x}$ consistent with the uncertainty principle? (15)

(e) What is the expectation value of $\hat{l}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$? (10)

Hints: (1) The Schrödinger equation for a state $|\Psi(x,t)\rangle$ of a system is

$$i\hbar\frac{\partial|\Psi(x,t)\rangle}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2|\Psi(x,t)\rangle}{\partial x^2} + V(x)|\Psi(x,t)\rangle .$$

(2) The standard deviation of any quantity A is defined as $\sigma_A = \sqrt{\langle\Psi|\hat{A}^2|\Psi\rangle - \langle\Psi|\hat{A}|\Psi\rangle^2}$.

(3) The special integral is $\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$.