University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Spring 2016

Paper D – Quantum Mechanics

Time allowed – 90 minutes

Instructions and Information:

- Answer both questions
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question is clearly indicated

Quantum Mechanics Basic Level

A hydrogen atom in the ground state is placed in a uniform magnetic field of strength $B_z = 0.55$ T. The Bohn magneton = $9.27400968(20)\times10^{-24}$ J/T.

(a) Compute the energy splitting of the two spin states of the hydrogen atom with respect to the z-direction of the magnetic field. (17)

(b) If you wish to excite with electromagnetic radiation (photons) the hydrogen atom from the lower to the higher energy state calculated above, what wavelength must the photon have? (18)

Quantum Mechanics Intermediate Level

A particle in the infinite square well has as its initial wave function which is an even mixture of the first two stationary states: $\Psi(x,0) = A\left[\psi_2(x) + \psi_3(x)\right]$, where

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), & \text{if } 0 \le x \le a, \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad E_n = \frac{\hbar^2\pi^2 n^2}{2ma^2}; \quad n = 1, 2, 3, \ldots$$

(a) Normalize $\Psi(x,0)$. (That is, find A. This is very easy, if you exploit the orthonormality of ψ_2 and ψ_3.) (10)

(b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time. To simplify the result, let $\omega \equiv \pi^2\hbar/2ma^2$. (15)

(c) Compute $\langle x \rangle$ in the state of $\Psi(x,t)$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (15)

(d) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? (10)

(e) Find the expectation value of $\hat{H}$. How does it compare with E_2 and E_3? (15)

(Hint: $\sin^2(x) = \frac{1}{2}\left(1 - \cos(2x)\right)$ and $\sin(x)\sin(y) = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$).