University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Fall 2009

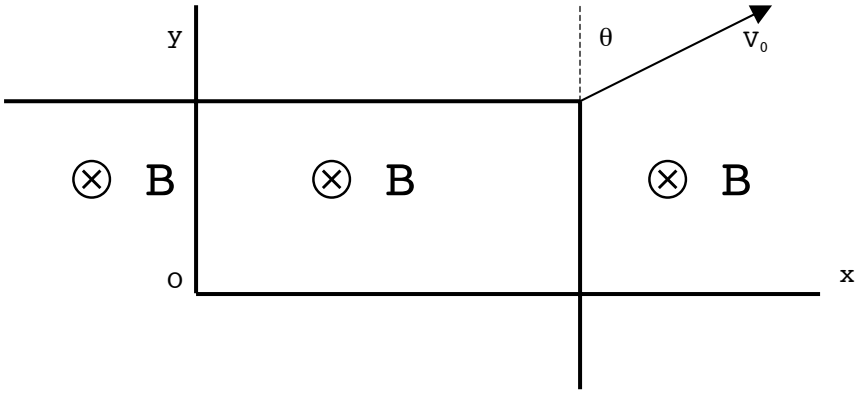
Paper B – Electromagnetism

Time allowed – 90 minutes

Instructions and Information:

- Answer both questions
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question are clearly indicated

1) Two straight conducting rails form a right angle where their ends are joined and lie along the +x, +y axes. An identical pair of rails moves with speed v_0 m/s along a line at θ degrees to the +y axis, as shown, so that a rectangular circuit is formed which increases in size. At t = 0 the area enclosed by the circuit is zero. A constant magnetic field, B Tesla, is directed into the plane of the paper.



(a) Obtain an expression for the area enclosed by the circuit at a time t (t > 0). (7)

(b) What is the expression for the magnitude of the magnetic flux through the circuit at time t ? (10)

(c) Use Faraday's law of induction to evaluate an expression for the magnitude of the induced emf at time t. (10)

(d) Calculate the magnitude of this emf when t = 10 s, B = 10^{-3} T, v_0 = 3 cm/s, θ = 45°. (5)

(e) Use Lenz's law to determine the direction of the induced current in the circuit, clockwise or anticlockwise, as seen by an observer on the +z axis. (3)

2) (a) State and explain with the aid of a diagram the Biot-Savart law. Be sure to label the diagram clearly and define all the variables you have used. (15)

(b) Use this law to show that the **B** field in free space at a point on the z-axis of a circular loop of wire of radius a, carrying a current I is given by,

$$\vec{B} = \frac{\mu_0 a^2 I}{2(z^2 + a^2)^{3/2}} \hat{z}$$

where the centre of the loop is the origin and z is the distance from the point to the plane of the loop. (28)

(c) A closed path is defined as the axis of the circular loop from z = –α to z = +α then back on the surface of the sphere of radius α. Calculate the line integral of **B** round this closed path in the limit of large α and show that this is compatible with the integral form of Ampere's law. (22)

$$\left[\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 (a^2 + x^2)^{1/2}} \right]$$