University of Louisville
College of Arts and Sciences

Department of Physics and Astronomy PhD Qualifying Examination (Part I)

Fall 2009

Paper D – Quantum Mechanics

Time allowed – 90 minutes

Instructions and Information:

- Answer both questions
- This is a closed book examination
- Start each question on a new sheet of paper – use only one side of each sheet
- Write your identification number on the upper right hand corner of each answer sheet
- You may use a non programmable calculator
- Partial credit will be awarded.
- Correct answers without adequate explanations will not receive full credit.
- Make sure your work is legible and clear
- The points assigned to each part of each question are clearly indicated

1) A small ant is confined inside a box of approximately 5-mm length and moves with a speed of about 1 mm/s. Consider the mass of the ant to be about 10^{-5} g.

(a) Treating the problem as a one-dimensional infinite square well potential, calculate the approximate value of the principal quantum number n associated with the energy levels. (17)

(b) What is the relative energy difference, defined by $\left(\frac{E_{n+1}-E_n}{E_n}\right)$, between two adjacent states for a particle in an infinite-square-well potential ? (11)

(c) For the specific value of n you calculated above, do you expect significant changes between the predictions of quantum mechanics for discrete energy levels and the predictions of Newtonian physics for continuous energy levels. Justify your answer. (7)

2) According to the postulates of quantum mechanics, for every physically measurable quantity A, to be called an observable, there corresponds a linear Hermitian operator $\hat{A}$ whose eigenvectors $\{|\psi_n\rangle\}$ form a complete basis. The measurement of an observable A may be represented formally by the action of $\hat{A}$ on a state vector $|\Psi\rangle$. The only possible result of such a measurement is given by one of the eigenvalues a_n of the operator $\hat{A}$. If the result of the measurement of A is a_n, corresponding to the eigenvector $|\psi_n\rangle$, then the state vector $|\Psi\rangle$ immediately after the measurement is $|\psi_n\rangle$. Before the measurement, the probability of the measurement of A being a_n is $P_n = |\langle\psi_n|\Psi\rangle|^2$.

Consider the following situation as an example. An operator $\hat{A}$, corresponding to an observable A, has two orthogonal and normalized eigenfunctions $|\psi_1^a\rangle$ and $|\psi_2^a\rangle$, with eigenvalues a_1 and a_2. An operator $\hat{B}$, corresponding to an observable B, has two orthogonal and normalized eigenfunctions $|\psi_1^b\rangle$ and $|\psi_2^b\rangle$, with eigenvalues b_1 and b_2. The eigenfunctions are related by

$$|\psi_1^a\rangle = (\frac{2}{\sqrt{13}}|\psi_1^b\rangle + \frac{3}{\sqrt{13}}|\psi_2^b\rangle), \text{ and } |\psi_2^a\rangle = (\frac{3}{\sqrt{13}}|\psi_1^b\rangle - \frac{2}{\sqrt{13}}|\psi_2^b\rangle).$$

(a) If A is measured and the value a_1 is obtained, what are the probabilities of a measurement of B obtaining the values b_1 and b_2, respectively ? (26)

(b) If B is measured as in part (a), then A is measured again, show that the probability of obtaining a_1 a second time is 97/169. (39)