1. How a Diffraction Grating works?

Diffraction gratings are optical components with a period modulation on its surface. Either the transmission (or the phase) changes in a periodic pattern. When a beam of light is incident on a diffraction grating, the optical power of the incident beam splits into several beams, each beam corresponding to a particular diffraction order. In the reflection side (the left side of the grating surface in the Figure below) the diffracted beams are given by:

\[ n_r \sin(\theta_r) = n_i \sin(\theta_{inc}) + m \frac{\lambda}{\Lambda} \]  

(1)

where \( m \) is an integer 0, ±1, ±2, ... All angles are measured against the normal to the grating surface, as indicated in the Figure. Rays
propagating the top side (above the normal to the surface) have positive angles, and rays propagating to the bottom side have negative angles.

Obviously, the incident and reflected beams propagate in the same medium, so we can write \( n_r = n_i \). In our experiments described below, we will have \( n_r = n_i = 1.00 \) for air.

For \( m = 0 \), the second term in the right hand side of Equation (1) does not bring any contribution and we get the usual law of reflection \( \theta_{r0} = \theta_{inc} \). For \( m \neq 0 \), the diffraction grating indeed bring a contribution, which can create a propagating diffraction order at an angle \( \theta_{rm} \) described by Equation (1).

On the transmission side (the right side of the grating in the Figure), the equation is similar:

\[
n_t \sin(\theta_t) = n_i \sin(\theta_{inc}) + m \frac{\lambda}{\Lambda} \tag{2}
\]

where \( m \) is an integer 0, ±1, ±2, ...

For \( m = 0 \), again, the second term in the right hand side of Equation (2) does not bring any contribution and we get the usual law of refraction as formulated in Snell’s law. For \( m \neq 0 \), the diffraction grating does bring a contribution, which can create a propagating diffraction order at an angle \( \theta_{tm} \) described by Equation (2). In our experiments here, we will have \( n_t = n_i = 1.00 \) for air.

2. Experiment:

In this experiment you will employ a He-Ne laser with a wavelength of 632.8 nm in vacuum. You will work with 3 different diffraction gratings, which are marked as 100, 300, 600 lines/mm. Choose initially the pattern with 600 lines-per-mm. Mount the grating on the
rotation stage with the grating grooves in the vertical direction so that the diffracted orders are displaced in a horizontal plane. Find the normal angle of reflection, i.e. the configuration where the incident beam is reflected straight back. Read this offset angle on the rotation station. All other angles that you measure later will be measured with respect to this offset angle. In other words, in all angle read-outs you will have to subtract this offset angle.

3. Finding the period of a diffraction grating:

Identify the diffraction order in reflection corresponding to \( m = -1 \). Starting from the normal incidence condition, rotate the rotation stage until the diffraction order \( m = -1 \) propagates straight back to the incident beam. The diffracted beam goes back to the laser cavity (or close to it). Read this angle on the rotation stage and subtract the offset angle. This configuration is known in the literature as Littrow’s configuration, and the measured angle \( \theta_{r-1} = -\theta_{inc} \) can be used to calculate the period of the grating by manipulating Equation (1) to give:

\[
\Lambda = \frac{\lambda_0}{2n_i \sin(\theta_{inc})}
\] (3)

Estimate your period of your grating and confirm if it agrees to the nominal value of the pattern. Remember: 600 lines-per-mm corresponds to a period of about 1.67 µm. Repeat your procedure for determining the grating period for the other two patterns.
4. Determining the angle of diffraction in transmission:

Now, consider the diffraction grating with a pattern of 100 lines/mm. Mount this diffraction grating on the rotation stage and rotate the stage to be at normal incidence, i.e., $\theta_{inc} = 0$, as represented in the Figure below.

Explore the diffracted beams in transmission on a screen at about 1 m away from the grating pattern. Measure the distance from the grating pattern to the observation screen, $\Delta z$. Next, at the observation screen, measure the transverse distance $\Delta y_m$ for each diffracted order with respect to the spot corresponding to the straight beam ($m=0$).

$$\theta_{t_m} = \arctan\left(\frac{\Delta y_m}{\Delta z}\right)$$  \hspace{2cm} (4)
Measure the diffraction angle for all orders, on both sides of the straight beam. Compare your results to the diffraction pattern that you would expect by the period of the grating you previously measured.

5. Measuring the resolving power of a diffraction grating at a particular order:

Still under the configuration described in the previous section (incident beam normal to the grating), measure for the order $m = 1$ the spot size of the diffracted beam at the observation screen. Find the angular width of this spot, $\Delta \theta_{t+1}$, which you can obtain by dividing the spot size by the distance $\Delta z$. Then, you can use Equation (5) to determine the smallest separation in wavelength that you can resolve with this grating at this specific diffraction order.

$$\Delta \lambda_0 = \frac{n_t \Delta \theta_{t+1} \lambda \cos(\theta_{t+1})}{m}$$  \hspace{1cm} (5)

6. Questions:

By starting from Equation (1) derive Equation (3). Also, starting from Equation (2) derive Equation (5). What can you do to improve the spectral resolving power of a grating?