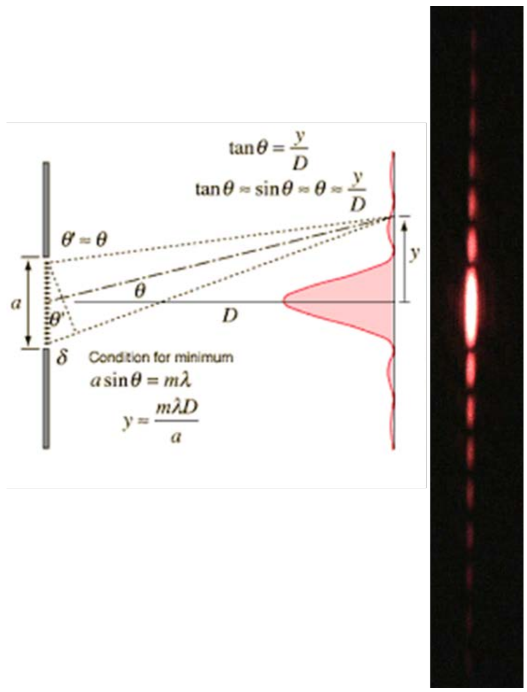
Fraunhofer diffraction

1. Diffraction by a slit

When a beam of light of wavelength λ arrives at a slit of width a, the diffracted light leaving the slit forms a pattern in space. As a function of angle θ the light intensity is given by



$$I(\theta) = I_o \left[\frac{sin(\beta)}{\beta}\right]^2 \qquad (1)$$

Where the parameter β is given by

$$\beta = \frac{\pi a\, sin(\theta)}{\lambda} \qquad (2)$$

Under the small angle approximation $sin(\theta)$ can be calculated from:

$$sin(\theta) \cong tan(\theta) = \frac{(y - y_0)}{D} \qquad (3)$$

where y is the transverse distance from the center at y_o, and D is the distance away from the slit.

Accordingly, the intensity will be zero where

$$\beta = m\,\pi \qquad (4)$$

$$(m = 1,\ 2,\ 3,\ \ldots)$$

since for these values $sin(\beta)$ will be zero.

We can calculate the position of the maxima by differentiating the expression for the intensity, and you can show that the maxima will occur where

$$tan(\beta) = \beta \qquad (5)$$

Solutions to this transcendental equation can be found with a calculator. They will be nearly equal to

$$\beta = (2m - 1)\,\frac{\pi}{2} \qquad (6)$$

$$(m = 1,\ 2,\ 3,\ \ldots)$$

except for the obvious one, where β is zero, corresponding to the center of the pattern.

2. The experiment

In this experiment you will illuminate a slit with light from a He-Ne laser of wavelength of 632.8 nm.

Measure the width of a slit

Place a slit on a holder in front of the laser beam (5-20 cm, this distance is not critical). Look for a diffraction pattern with a white screen at about 1 meter away from the slit. You should see at the observation screen a bright spot in the middle that is surrounded by dark bands and adjacent bright spots that are weaker and weaker as they are away from the center peak. This pattern is mathematically described by Equation (1), which is known a sinc-square function. Measure with good precision the distance D between the slit and the white screen. Measure the location of the dark bands, considering that the center of the bright peak is defined as $y_0 = 0$. Each dark band corresponds to an integer value of m in Equation (4). You should measure as many dark bands as you can visualize. In your report, make a table of m versus y_m. From each value of y_m, determine $sin(\theta_m)$ from Equation (3). Create a plot of $sin(\theta_m)$ versus m. You should get approximately a straight line. From the slope of this curve, you can use Equations (2) and (4) to determine the width of the slit.

Compare with the theory

In addition to the position of the dark bands, measure the location of the bright peaks. In your report, now that you know the width of the slit, use Equation (2) to calculate experimental values of β for each of the sequential minima and maxima. Compare those results with the theoretical values. In the case of the maxima, this will require that you solve the transcendental Equation (5) for β for the first solutions greater than zero.

Diffraction pattern of an adjustable slit and a circular slit

Replace the fixed slit with an adjustable slit, which you can change the width and rotate around an axis parallel to the beam of light. First, change the slit width and observe the diffraction pattern. Then, for a fixed width, rotate the slit around the optical axis and observe the diffraction pattern. Report your findings of both observations. Take a circular slit (a pinhole) in front of the laser beam, and report the diffraction pattern you obtain.

3. Use diffraction to measure the diameter of your hair

Instead of the slits, mount a piece of hair in a frame and place it in front of the laser beam. According to Babinet's principle the pattern is similar to the complementary pattern of a slit. Measure the positions of the first zeros (dark bands) on each side of the central maximum. Calculate the width of the hair, as you did for the slit.

4. Computing a Fraunhofer pattern

Use a personal computer and your preferred software to plot the theoretical results for intensity $\frac{I}{I_o}$ versus $(y - y_o)$, as described by Equations (1)-(3). Make sure to enter the values that you measured for the slit width a and wavelength λ. Compare this calculation with your experiment. Include a screen dump of this graph in your lab report.

5. Diffraction depends on wavelength

It is instructive to consider the diffraction pattern for different colors, since the wavelength dependence of diffraction is immediately obvious. What would happen to the diffraction pattern if you change the laser beam from red (633 nm) to the yellow, green and blue colors?