Planar dielectric waveguides

Abstract:

An optical waveguide is a physical structure that guides electromagnetic waves in the optical spectrum. They are used as components in integrated optical circuits, as the transmission medium in long distances for light wave communications, or for biomedical imaging. We can classify the waveguide according to different methods. According to the structures: planar, strip, or fiber waveguides, mode structure: single-mode, multi-mode, refractive index distribution: step or gradient index and material: glass, polymer, semiconductor. Here we discuss the planar dielectric waveguide specifically from the configuration, waveguide mode, field distribution, dispersion relation and group velocity aspects.

1: Configuration

Fig. 1 shows the configuration of a typical planar dielectric waveguide. A slab of dielectric material, called film or core, surrounded by media of lower refractive indexes, called cover and subtract as the upper and lower, respectively.

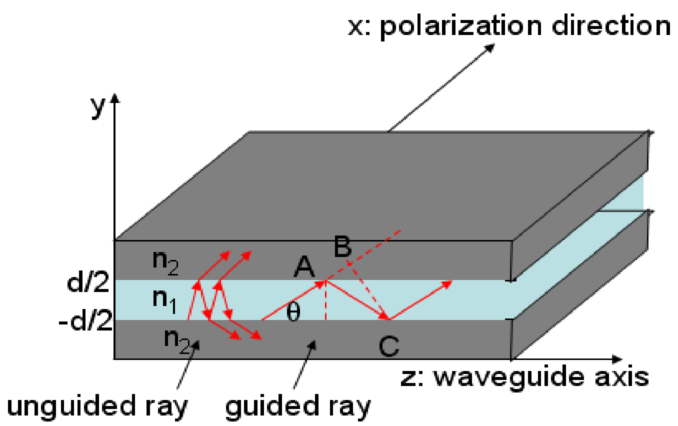

Fig.1 (Planar dielectric waveguide configuration. The width of the slab is d and refraction index is n_1, and the cover and subtract have same refraction index n_2.)

A light ray can be guided inside the slab by total internal reflection in the zigzag fashion. Only certain reflection angle θ will constructively interfere in the waveguide and hence only certain waves can exist in the waveguide (this will be discussed more in section 2 waveguide modes).

Case 1: θ smaller than complement of the critical angle

$$\theta < \bar{\theta}_c = \pi/2 - \sin^{-1}(n_2/n_1) = \cos^{-1}(n_2/n_1) \qquad \text{case 1}$$

Total internal reflection will happen at the boundaries. Then the rays can travel in z direction by bouncing between the slabs surfaces without loss of energy (figure showed in the right of Fig.1). And we also assume that all the materials are lossless.

Case 2: θ larger than complement of the critical angle

$$\theta > \bar{\theta}_c = \pi/2 - \sin^{-1}(n_2/n_1) = \cos^{-1}(n_2/n_1) \qquad \text{case 2}$$

Total internal reflection can not happen at the boundaries. Then rays will lose a portion of their power at each reflection, and eventually they will vanish.

In this paper, we only consider symmetric planar dielectric waveguide, which is the cover and subtract have the same refraction index.

2: Waveguide modes

Because only certain reflection angle θ are allowed. We need self consistency condition to find θ_m which can survive in the waveguide, and m means mth mode.

Assumption

The field in the slab is in the form of a monochromatic TEM plane (Electric wave is oscillating perpendicular to incident and reflection plane, here is x direction), and wave bounces in case 1 situation discussed in section of configuration.

$$\lambda = \lambda_0 / n_1, c_1 = c_0 / n_1$$

The wave vector is $n_1 K_0$, having $K_x = 0, K_y = n_1 K_0 \sin\theta, K_z = n_1 K_0 \cos\theta$.

Self-consistency condition: the phase shift between the two waves must be 0 or a multiple of 2π

A wave should reproduce itself after each round trip, otherwise they will have phase shift not equal to a multiple of 2π. In one round trip, the twice reflected wave lags behind the original wave by a distance $AC - AB = 2d * \sin\theta$, as in FIG 1. At the dielectric boundary, each internal reflection will introduce a phase ϕ_r.

$$\frac{2\pi}{\lambda} 2d \sin\theta - 2\phi_r = 2\pi m \qquad \text{Eq 1}$$

ϕ_r, depends on the angle θ and the polarization of the incident wave. According to the TE wave reflection phase shift and $\theta_1 = \pi/2 - \theta, \theta_c = \pi/2 - \bar{\theta}_c$, θ_1 is the complement angle of θ.

$$\tan\frac{\phi_r}{2} = \sqrt{\frac{\sin^2\bar{\theta}_c}{\sin^2\theta} - 1} \qquad \text{Eq 2}$$

As θ varies from 0 to $\bar{\theta}_c$, ϕ_r varies from π to 0. Substitute Eq2 into Eq1 and we can get self consistency condition for TE modes.

$$\tan(\pi \frac{d}{\lambda}\sin\theta - m\frac{\pi}{2}) = \sqrt{\frac{\sin^2 \overline{\theta_c}}{\sin^2 \theta} - 1} \qquad \text{Eq 3}$$

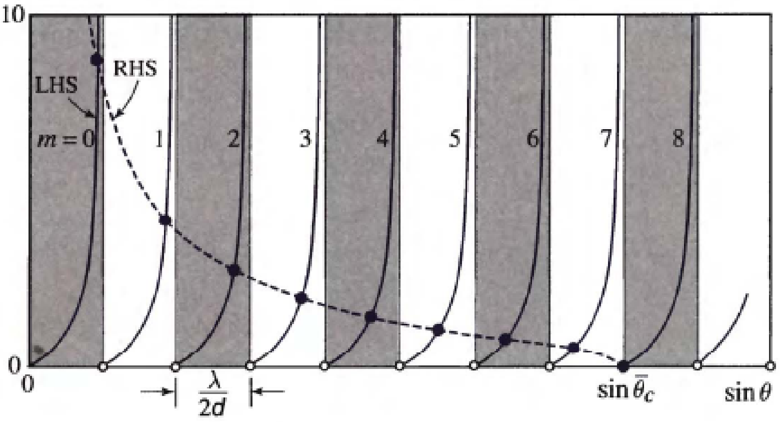

Fig 2: (Graphical solution of Eq3 to determine the bounce angle θ_m of the modes of a planar dielectric waveguide. The RHS and LHS are plotted versus sinθ. The intersection points, marked by filled circles , determine $\sin\theta_m$. Each branch of the tan or cot function in the LHS corresponds to a mode. In this plot $\sin\overline{\theta}_c = 8(\lambda/2d)$ and the number of mode is M=9.)

Propagation constant

The wave vector with angle θ_m have the components $(0, K_y = n_1 K_0 \sin\theta_m, n_1 K_0 \cos\theta_m)$. The z component is the propagation constants. It shows in Fig. 3

$$\beta_m = n_1 K_0 \cos\theta_m \qquad \text{Eq 4}$$

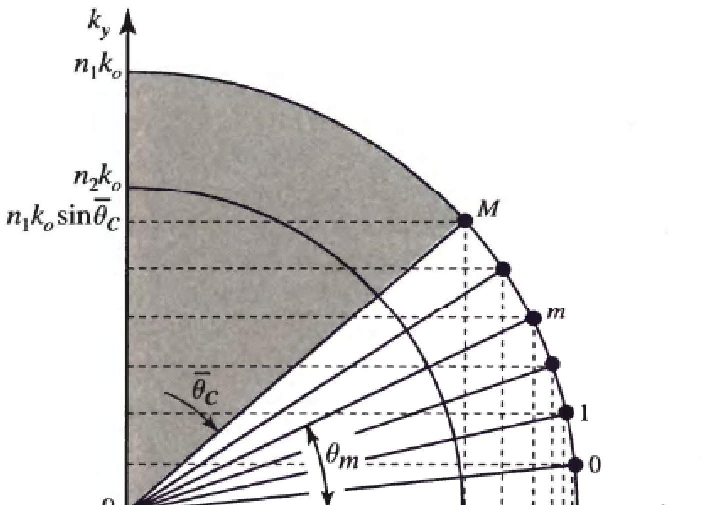

Fig. 3 (The bounce angles θ_m and the corresponding components of the wave vector of the

waveguide modes are indicated by dots. The propagation constant lies between n_2K_0 and n_1K_0.)

Number of modes

According to Eq3 and because $\sin\theta \le \sin\bar{\theta}_c$, we can get the number of waveguide's modes.

$$\bar{M} = \frac{\sin\bar{\theta}_c}{\lambda/2d};$$

$$\bar{M} = \frac{2d}{\lambda_0} NA; \qquad \text{Eq 5}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

NA is the numerical aperture. And the number of mode is increased to the nearest integer. In a dielectric waveguide there is at least one TE mode, since the fundamental mode m=0 is always allowed.

Single mode waveguide: when $\lambda/2d > \sin\bar{\theta}_c$, only one mode is allowed. This occurs when the slab is thin enough or the wavelength is sufficiently long. At this situation, there is no cutoff wavelength. But each other mode, higher than 0, has cutoff wavelength.

The condition for single mode operation is that $\nu > \nu_c$,

$$\nu_c = \omega_c / 2\pi = \frac{1}{NA}\frac{c_0}{2d} \qquad \text{Eq 6}$$

This can be shown in Fig 4.

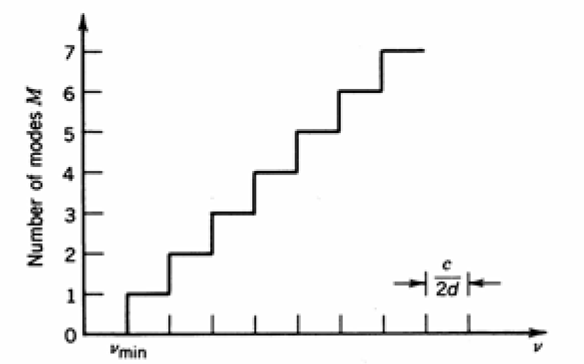

Fig 4 (Number of TE modes as a function of frequency.)

3 Field distribution

Internal field

There are two composed TEM plane waves traveling at angles θ_m and -θ_m with z axis with wave vector components $(0, \pm n_1 K_0 \sin\theta_m, n_1 K_0 \cos\theta_m)$. They have the same amplitude and phase shift mπ at the center of the slab then we can get electric field complex amplitude

$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z) \qquad \text{Eq 6}$$

β_m is propagation constant., a_m is a constant, and

$$u_{m(y)} \propto \begin{cases} \cos(2\pi \dfrac{\sin\theta_m}{\lambda} y), m = 0,2,4..... \\ \sin(2\pi \dfrac{\sin\theta_m}{\lambda} y), m = 1,3,5...... \end{cases}, -d/2 \le y \le d/2 \qquad \text{Eq 7}$$

Note: the field does not vanish at the boundary. If the interface of the boundaries are mirrors, then the external field are zero.

External field

The external field must match the internal field at all the boundary points y= $\pm$d/2. So it must vary with z as $\exp(-j\beta_m z)$. Substitute $E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z)$ into Helmholtz equation $(\nabla^2 + n_2^2 K_0^2)E_x(y,z) = 0$, we can get

$$u_{m(y)} \propto \begin{cases} \exp(-\gamma_m), y > d/2 \\ \exp(\gamma_m), y < -d/2 \end{cases}; \qquad \text{Eq 8}$$

$$\gamma_m = n_2 K_0 \sqrt{\frac{\cos^2\theta_c}{\cos^2\theta_1} - 1}$$

γ_m is the extinction coefficient. And this wave is called evanescent wave.

As the mode number m increases, θ_m increases, and γ_m decreases. Higher order modes therefore penetrate deeper into the cover and substrate. It shows in Fig. 4

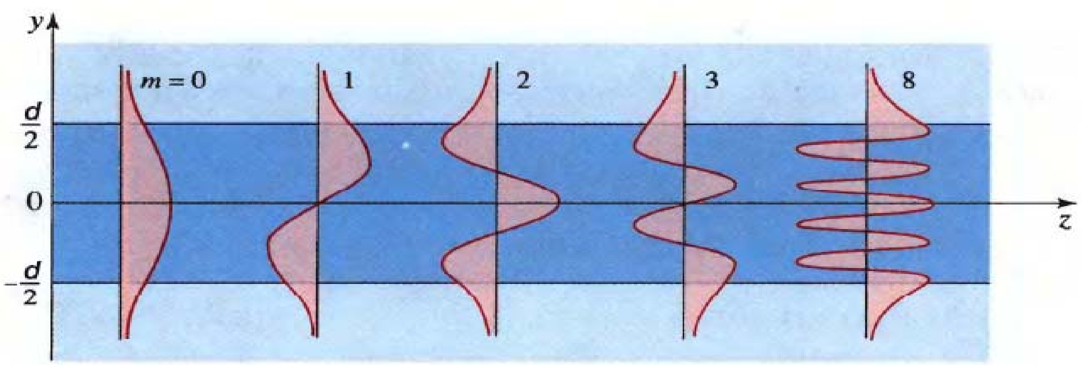

Fig 4 (Field distribution for TE guided modes in a dielectric waveguide.)

The field distribution of the lowest order TE mode m=0 is similar in shape to that of the Gaussian beam, but guided light does not spread in the transverse direction as it propagates in the axial direction. In a waveguide, the tendency of light to diffract is compensated by the guiding action of the medium. It shows in Fig. 5

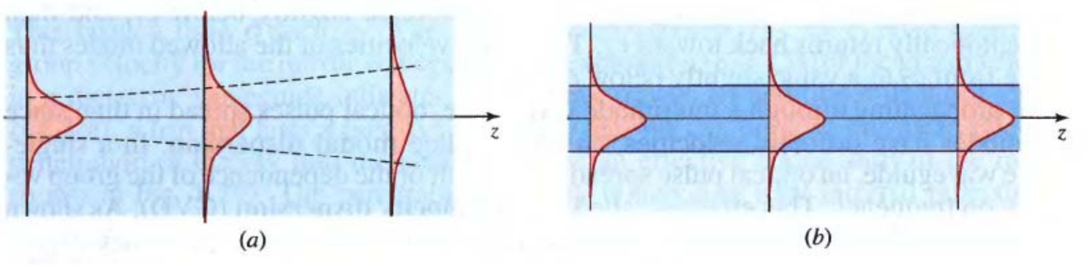

Fig. 5 (a) Gaussian beam in a homogeneous medium; b) Guided mode in a dielectric waveguide.)

4 Dispersion relation and group velocities

Change Eq1 in terms of β and m using the function that $K_y^2 = (\omega / c_1)^2 - \beta^2$

$$2d\sqrt{\frac{\omega^2}{c_1^2} - \beta^2} = 2\phi_r + 2\pi m \qquad \text{Eq 9}$$

Also Eq2's form can be changed to the dispersion relation.

$$\tan^2\left(\frac{d}{2}\sqrt{\frac{\omega^2}{c_1^2} - \beta^2} - m\frac{\pi}{2}\right) = \frac{\beta^2 - \omega^2 / c_2^2}{\omega^2 / c_1^2 - \beta^2} \qquad \text{Eq 10}$$

Rewrite Eq10 into parametric form,

$$\frac{\omega}{\omega_c} = \frac{\sqrt{n_1^2 - n_2^2}}{\sqrt{n_1^2 - n^2}}\left(m + \frac{2}{\pi}\tan^{-1}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2 - n^2}}\right), \beta = n\omega / c_0,$$

$$\omega_c / 2\pi = c_0 / 2dNA \qquad \text{Eq 11}$$

n is the effective refractive index defined in Eq11, and ω_c is the mode cutoff angular frequency.

We can find out the effect of a stronger confinement of waves of shorter wavelength in the medium of higher refractive index. We should note that higher order modes travel longer distance in the waveguide than do lower order modes. Thus for light launched at the same time, the time of arrival at the far end of the wavelength will depend on the path taken. This results in a spread in time of arrival. This is pulse broadening.

Modal dispersion:

In propagation through a multimode waveguide, optical pulses spread in time since the modes have different velocities.

Group velocity dispersion (GVD):

The group velocity is obtained from the dispersion relation by determining the slope $v = d\omega / d\beta$ for each of the guided modes. And the group velocity of the allowed modes range from c_2 to a value slightly below c_1.

In a single mode waveguide, an optical pulse spreads as a result of the dependence of the group velocities on frequency. It happens in homogeneous materials by virtue of the frequency dependence of the refractive index of the material.

Moreover, it occurs in waveguides even in the absence of material dispersion. With a source with a range of wavelengths, there will be a range of group velocities. It results from the guiding properties of the waveguide and has nothing to do with the frequency dependence of the refractive index. Longer wavelength has more energy in the cladding and thus travels faster.

Each mode has a particular angular frequency at which the group velocity changes slowly with frequency—the point at which v reaches its minimum value so that its derivative with respect to ω is 0. At this frequency, the GVD coefficient is 0 and pulse spreading is negligible.