Derivation of Oort's Constants

Univ. Louisville, 6 Mar 2014

19.5 Differential Galactic Rotation

Galactic bulge rotates like solid body

Sun orbits Galaxy - must be Keplerian, because Galactic mass concentrated in bulge LSR (Local Standard of Rest), on small scale, reflects uniform motion of stars in Solar neigh-

borhood.

At SOME point, must see differential rotation

Recall that $v \propto r^{-1/2}$ for Keplerian motion however, $v = \omega r \propto r$ for solid body rotation

Derivation of Oort's Constants for Differential Rotation

based on work in 1927 by Jan Oort (1900-1992).

Assume circular, co-planar orbits. Equation/figure numbers correspond to Ryden & Peterson (2010).

See Fig. 19.15 and the figure I drew in pdf format, which has two triangles: one for position, and one for velocity.

Assume stars orbit in concentric, coplanar circles. We note a star with quantities:

 R_0 = distance from Sun to Gal Ctr R = distance from star to Gal Ctr d = distance from Sun to star Θ_0 = Sun's circular speed Θ = star's circular speed $\omega_0 = \Theta_0/R_0$ LSR angular velocity around Galaxy $\omega = \Theta/R$ star's angular velocity around Galaxy ℓ = Gal longitude of star α = angle from line of sight (Sun to star) to star's velocity

There are also observables:

 v_r = star's radial velocity, measured from Earth, corrected to a sun-centric or LSR reference μ = star's proper motion in arcsec/yr, measured from Earth, corrected to a sun-centric or LSR reference

These parameters are usually extremely well: ℓ , v_r

These parameters are usually fairly well, depending on the sample we use to determine the LSR and the center of the Galaxy: R_0 , Θ_0 (and thus ω_0)

These parameters are usually known somewhat, but generally less accurately the more distant a star is: d, μ . The Hipparcos and GAIA satellites have <u>revolutionized</u> our knowledge of stellar

distances and proper motions.

Reddening/extinction is a challenge, and μ can be immeasurably small on the span of a year, thus we sometimes must use a baseline of several years to decades to measure μ .

These parameters are generally unknown: $R, \Theta, \omega, \alpha$ We wish in the end to know $\omega(r)$ to understand Galactic rotation.

star's radial velocity (from Sun) is $\Theta \cos \alpha$ LSR velocity along line of sight to star is $\Theta_0 \sin \ell$ So star's radial velocity wrt LSR is $v_r = \Theta \cos \alpha - \Theta_0 \sin \ell$ (Eqn 19.33)

Law of Sines gives: $\sin \ell/R = [\sin(90 + \alpha)]/R_0 = [\cos \alpha]/R_0$ (Eqn 19.34) or $R_0 \sin \ell = R \cos \alpha$

Using $\omega_0 = \Theta_0/R_0$, $\omega = \Theta/R$ gives

 $v_r = \omega R \cos \alpha - \omega_0 R_0 \sin \ell$ $= \omega R_0 \sin \ell - \omega_0 R_0 \sin \ell$

This yields the first Oort Equation, for radial velocity: = $(\omega - \omega_0)R_0 \sin \ell$ (Eqn 19.37)

For rigid rotation, $\omega = \omega_0$ and $v_r = 0$ For differential rotation, $\omega \neq \omega_0$ and $v_r \neq 0$

To find v_t : calculate the difference between Sun's star's tangential velocity (from Sun) is $\Theta \sin \alpha$ LSR velocity \perp line of sight to star in Gal plane is $\Theta_0 \cos \ell$ So star's tangential velocity wrt LSR is $v_t = \Theta \sin \alpha - \Theta_0 \cos \ell$ (Eqn 19.38)

from Law of Sines: $\sin \ell / R = [\sin(90 - \ell - \alpha)]/d = [\cos(\ell + \alpha)]/d$

use $\cos(x+y) = \cos x \cos y - \sin x \sin y$ to get $\sin \ell/R = (\cos \alpha \cos \ell - \sin \alpha \sin \ell)/d$

Solve for $\sin \alpha$: $\sin \alpha = [d/\sin \ell][(\cos \alpha \cos \ell)/d - (\sin \ell/R)]$ $= (\cos \alpha \cos \ell / \sin \ell) - (d/R)$ Remember $\sin \ell / R = \cos \alpha / R_0 - \cos \alpha = (R_0/R) \sin \ell$

Plugging above two, $\omega = \Theta/R$, $\omega_0 = \Theta_0/R_0$ into Eqn 19.38 gives:

 $v_t = \Theta[(\cos\alpha \cos\ell/\sin\ell) - (d\sin\ell/R)] - \Theta_0 \cos\ell$ = $R\omega[(\cos\alpha \cos\ell/\sin\ell) - (d/R)] - R_0\omega_0 \cos\ell$ = $R\omega[(R_0/R) \sin\ell \cos\ell/\sin\ell - (d/R)] - R_0\omega_0 \cos\ell$ = $\omega R_0 \cos\ell - \omega dR_0\omega_0 \cos\ell$

This yields the second Oort Equation, for tangential velocity: = $R_0(\omega - \omega_0) \cos \ell - \omega d$ (Eqn 19.42)

The OORT FORMULAE are Eqns 19.37, 19.42 for v_r , v_t

For Solar neighborhood, $d \ll R$ and we can make some approximations

Using Taylor expansion, $\omega - \omega_0 \approx (d\omega/dR)|_{R_0}(R - R_0)$ (Eqn 19.43)

We define Oort ct $A \equiv -(R_0/2)(d\omega/dR)_{R0}$ (Eqn 19.48)

which makes $v_r = -2A(R - R_0)\sin\ell$

For small d (meaning small $90^{\circ} - \ell - \alpha$), make d hypotenuse of triangle of angle ℓ and one side $\approx R - R_0$. Then $R - R_0 \approx d \cos \ell$ Use $\sin 2\ell = 2 \sin \ell \cos \ell$ to get $v_r = -2A[d \cos \ell] \sin \ell$ then $v_r = Ad \sin(2\ell)$ (Eqn 19.47)

For the tangential cpt v_t (Eqn 19.42), using the same approximations $\omega - \omega_0 \approx (d\omega/dR)|_{R0}(R - R_0) \approx (d\omega/dR)|_{R0}(-d\cos\ell)$ and $R - R_0 \approx d\cos\ell$ and ignoring terms d^2 and higher and using $\cos^2 \ell = (1/2)(1 + \cos 2\ell)$, is $v_t = R_0(\omega - \omega_0)\cos\ell - dw$ $= R_0 \frac{d\omega}{dR}|_{R0}(R - R_0)\cos\ell - d\omega$ $= R_0 \frac{d\omega}{dR}|_{R0}(-d\cos\ell)(\cos\ell) - d\omega$ $= 2Ad(1/2)(1 + \cos 2\ell) - d\omega$ $= d[A + A\cos 2\ell + (\omega_0 - \omega) - \omega_0]$ $= d[A + A\cos 2\ell + \frac{d\omega}{dR}|_{R0}(-d\cos\ell) - \omega_0]$ Let $d^2 - > 0$, then $v_t = d[A + A\cos 2\ell - \omega_0]$ Define $B \equiv A - \omega_0$ (Eqn 19.50) giving $v_t = d(A\cos(2\ell) + B)$ (Eqn 19.49)

WHEW!

If we plot $v_r(\ell)$ we see *double sinusoid* (period = 180° in ℓ)

See Figs 19.16, 19.17

Stars with $R < R_0$ pass us up/lap us. Stars with $R > R_0$ are passed up/lapped by us. Since ℓ is measured from Gal Ctr, BOTH situations lead to higher ℓ

19.6 Characterizing the Rotation Curve (at least near the LSR)

 $\omega_0 = \Theta_0 / R_0 = A - B$ (in Oort cts)

From observations: Oort ct A = 14 - 15 (km/s)·kpc, from B-stars, Cepheids (explain) Oort ct $B \approx -10 - - 12$ (km/s)·kpc

So $\omega_0 = \Theta_0 / R_0 = A - B = 25 - 26 \ (\text{km/s}) \cdot \text{kpc}$

We can get AR_0 from radio. Latest results: ESO/VLT, tracing orbit of star S2 around BH in Gal. Ctr, using Kepler's 3rd Law, is $R_0 = 7.94 \pm 0.42$ kpc (Eisenhauer et al. 2003, ApJ)

 $\Theta_0 \approx 220 \text{ km/s}$

What SHOULD Oort's cts be?

Derive Oort's Cts for LSR in Keplerian orbit about $M_{Gal} = 1.5 \times 10^{11} M_{\odot}$:

$$\begin{split} &\omega^2 R = G M_g / R^2 \ (\text{F=ma in circular orbit}) \\ &\text{or} \\ &\omega(R) = (G M_G / R^3)^{1/2} \ (\text{Eqn A}) \\ &d\omega / dR = -(1.5) (G M_G)^{1/2} R^{-5/2} = -3\omega / 2R \\ &\text{so } A = -(R_0 / 2) (d\omega / dR)_{R0} \\ &A = (-R_0 / 2) (-3\omega_0 / 2R_0) \\ &A = (3/4) \omega_0 \end{split}$$

Plug in values: $\omega_0 R_0 \approx 220 \text{ km/s}$ $\omega_0 \approx 220/7.94 = 27.7 \text{ km s}^{-1} \text{ kpc}^{-1}$ So $A = (3/4)\omega_0 \approx 21 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $B \equiv A - \omega_0 = -(1/4)\omega_0$ $B = -6.9 \text{ km s}^{-1} \text{ kpc}^{-1}$

BUT we observe A = 14 - 15 and B = -12 - - 10 in same units – WHY? Galaxy is NOT a point mass!

Rotation Curve of Galaxy

 $\Theta(R)$ is rotation curve of Galaxy

We can measure for other galaxies via HI emission, but for the Galaxy it's harder. We can measure rotation curves across several kpc of distance, interior to the solar circle (the region inside the sun's orbit) by looking at molecular or H I emission from clouds of gas along a line of sight. The problem is that determining their distance and proper motion is very hard. So, we use a statistical approach.

We note that the maximum v_r occurs when line of sight touches tangent of a star's orbit: $R_{min} = R_0 \sin \ell$ (related to Eqn 19.56)

Geometrically, we can invert Eqn A at the tangent point (R_{min}) for the velocity: $\Theta(R_{min}) = v_{r,max} + \Theta_0 \sin \ell$ then divide through by $R = R_0 \sin \ell$ and get the angular velocity $\omega(R_{min}) = \omega_0 + [v_{r,max}/(R_0 \sin \ell)]$ (Ean 19.58)

where we observe $v_{r,max}$ and we can determine $R_{min} = R_0 \sin \ell$ observationally, too. NB: We need any two of R_0 , Θ_0 , ω_0

If we use $v_r = -2A(R - R_0) \sin \ell$ and assume $d \ll R_0$ (nearby objects) then we get (after some algebra)

 $v_{r,max} = 2AR_0(\sin \ell)(1 - \sin \ell)$ so we get AR_0

SO WE CAN WORK OUT ROTATION VELOCITY AS A FUNCTION OF RADIUS

We best observe it via radio/microwaves (HI/CO emission), because it probes to bigger distances than stars.

HI/CO traces spiral arms - so only a few places with v tangent to line of sight

RESULTS: Rotation curve NOT Keplerian! Rigid body rotation in Gal Bulge Then rotation curve (v(R)) minimizes at 3kpc, then RISES SLOWLY Rot'n curve RISES MORE beyond Sun!! Thus there is lots of mat'l outside of Solar orbit - just as much as inside!

See Fig 19.19

For radii outside the solar circle, we need to use objects for which we can determine distance e.g. Cepheid variables.